LCLS-II Technical Note

Inducing a Positive Chirp in a Beam using Two Bunches + a Dechirper

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Inducing a Positive Chirp in a Beam using Two Bunches + a Dechirper

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Summary

- Normally the wakefield of a short bunch induces a negative chirp, *i.e.* the front of the bunch feels nearly no wake while the back experiences energy loss. We here look into the question of whether, by using two bunches and a dechirper, we can induce a positive chirp in the second bunch

- The longitudinal wake of a dechirper varies approximately as $-\cos ks$, with s the distance behind the driving bunch; the first maximum positive slope occurs at $s = \pi/(2k)$. With a 1 kA driving bunch we would like a chirp of $h \gtrsim 1$ MeV/fs.

- We find this level of h using a dechirper of length $L = 1$ m, half aperture $a = 0.25$ mm, corrugation size ~ 10 μm, and with a bunch spacing $\Delta T \sim 200$ fs. The dechirper may be difficult to build.

- The tranverse wakes are very strong. However, the transverse jitter tolerance for the first bunch can be made to be ~ 1 μm.

- A metallic pipe lined with a ~ 10 μm thick layer of dielectric can also be used to induce the positive chirp

Introduction

• When a short bunch passes by a vacuum chamber object, such as a dechirper, it will induce a wakefield resulting in a negative chirp, *i.e.* the front of the bunch will see almost no wakefield while the back of the bunch loses energy

• Since the wakefield of a dechirper over the longer time scale oscillates, one might consider an arrangement where a leading driving bunch excites a dechirper and a trailing bunch follows at the right distance to obtain a positive chirp

• Here I will do some rough calculations to see how feasible the idea is

• Let us assume: the trailing bunch has a charge $Q = 100$ pC and peak current $\hat{I} = 2$–3 kA (or length $t_{fwhm} \approx 33$–50 fs); we would like an induced chirp $h \sim +1$ MeV/fs; ideally the bunch spacing should be in the range $\Delta T \sim 10$–50 fs

Note: for $\hat{I} \lesssim 1$ kA, such chirp correction is not needed–Yuantao

Introduction Cont'd

• The trailing bunch induces its own wake with a negative chirp over the bunch. Here we will include the effect of both bunches

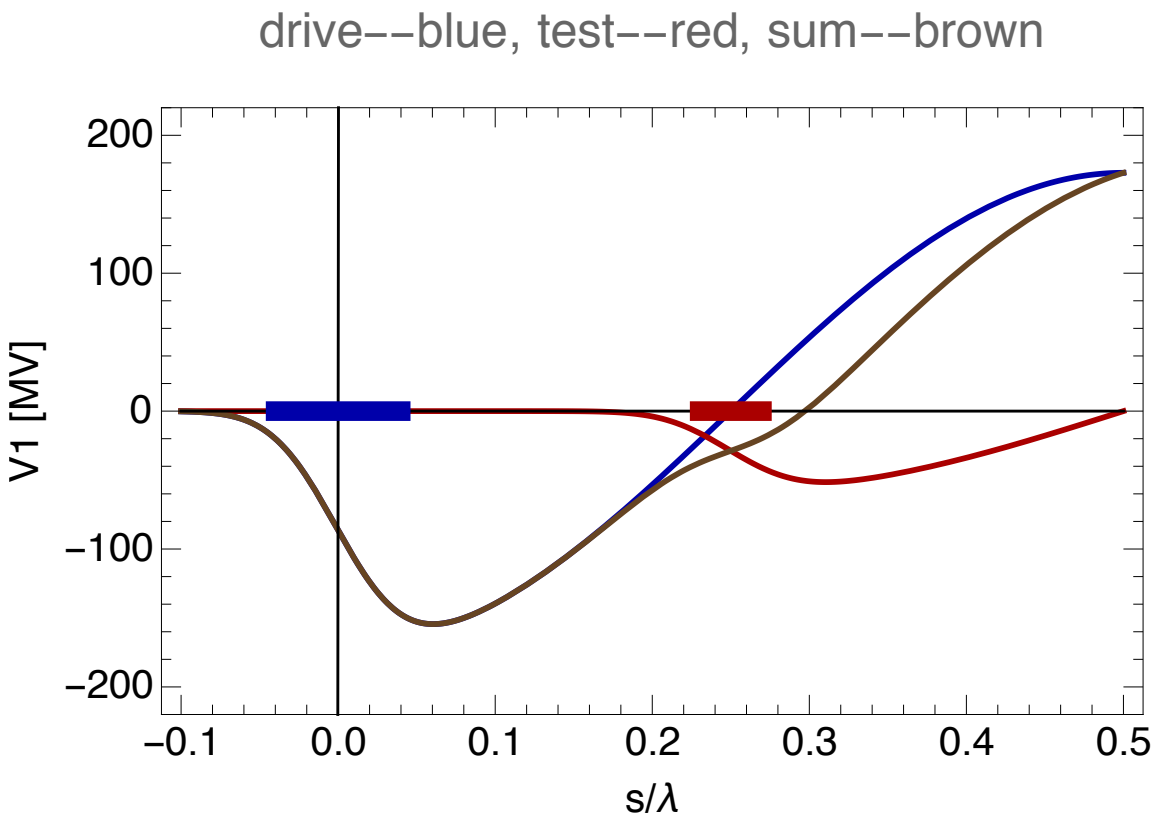
• In the study of wakefield acceleration, it was discovered that the *transformer ratio* R of a driving bunch with a symmetric longitudinal distribution is limited to 2. That is, the ratio of the peak energy gain behind the bunch to the average energy loss of the bunch is 2 or less. [Bane, Chen, Wilson, IEEE Trans.Nucl.Sci. 32 (1985) 3524].

For a short driving bunch, *i.e.* $k\sigma_z$ small (k is wave number of wake oscillation, σ_z is bunch length), $R = 2$

• We will see that the same kind of thing applies to the chirp, *i.e.* the chirp induced behind a short driving bunch is limited in amplitude to twice the chirp within the bunch. But a factor of 2 is enough to guarantee a net positive chirp (at a proper bunch separation) for two identical bunches

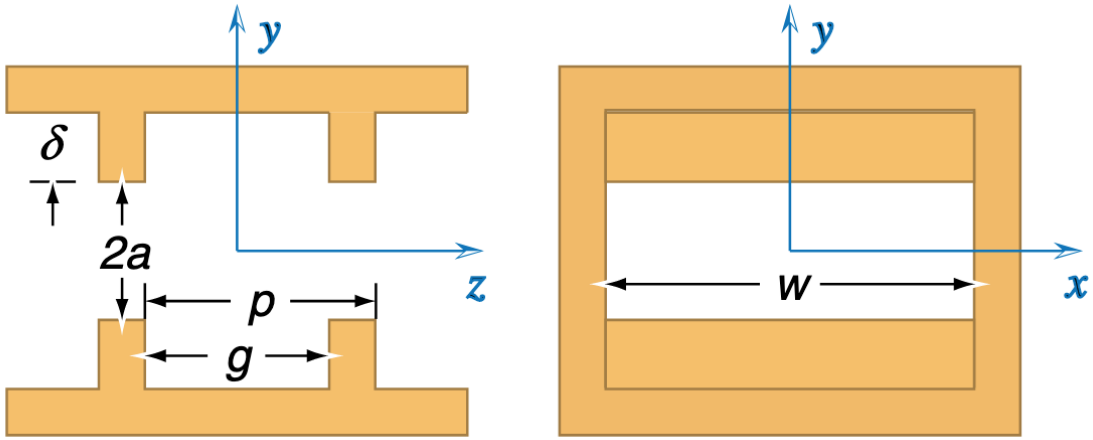
Sketch of Idea

• The wakes of both the leading and trailing particles need to be included. The self-wake of the trailing particle will tend to counteract that of the leading particle



Sketch of induced voltage in two-bunch configuration. The leading bunch's wake is in blue, the trailing bunch's in red, and the sum is given in brown. The bunch positions and lengths are indicated by the flat rectangles, with their heads to the left

Dechirper Wake



Longitudinal and transverse view of a rectangular dechirper, showing full aperture 2a, period p, gap g, depth δ, and width w

• A dechirper, like the RadiaBeam/SLAC dechirper, is a corrugated, metallic pipe, with δ, p, $g \lesssim a$ and (*required*) $\delta \gtrsim p$

• Consider first a round dechirper, with a the iris radius. The wakefield $w(s) \approx 2H(s)\kappa_r \cos ks$, with loss factor $\kappa_r = Z_0 c/(2\pi a^2)$ and wave number $k_r = \sqrt{2p/(g\delta a)}$; $Z_0 = 377\ \Omega$, $H(s) = 0$ (1) for $s < 0$ (> 0)

• For a flat dechirper, like the RadiaBeam/SLAC dechirper, loss factor $\kappa_f = \frac{\pi^2}{16} Z_0 c/(2\pi a^2)$ and wave number $k_f = \sqrt{p/(g\delta a)}$

Dechirper Wake Cont'd

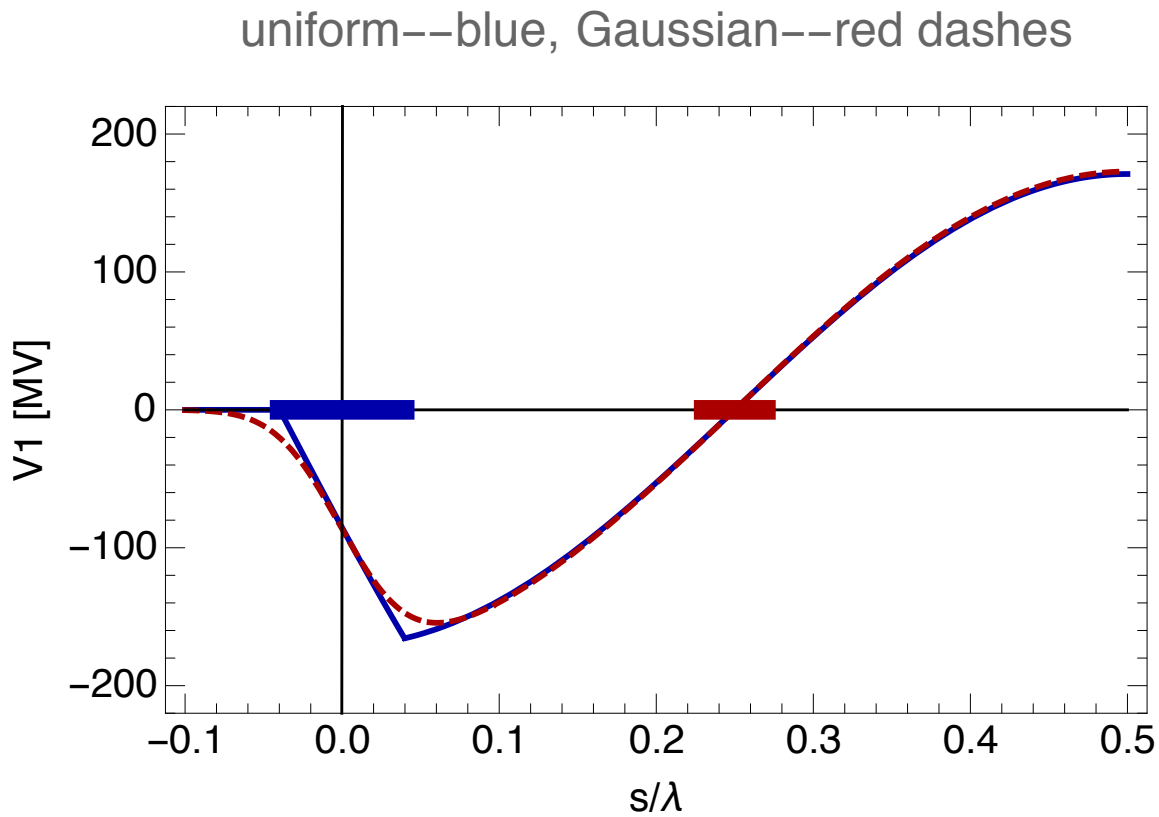
- Consider the parameters of the RadiaBeam/SLAC dechirper, with half-aperture $a = 0.7$ mm, where $p = 0.5$ mm, $g = 0.25$ mm, and $\delta = 0.5$ mm. The frequency is $f = 110$ GHz and the optimal bunch spacing of $\Delta T = \frac{1}{4}\lambda/c = 2200$ fs

The loss factor $\kappa_f = 23$ kV/(pC m); assuming leading bunch charge $Q_1 = 300$ pC, structure length $L = 1$ m, then the chirp at the second bunch (ignoring self-wake for the moment) $h = 2e\kappa_f c Q_1 kL = 0.01$ MeV/fs, way too small

- We need a much higher frequency. Consider: $a = 0.25$ mm, $p = 25$ μm, $g = 6.25$ μm, $\delta = 25$ μm; $Q_1 = 300$ pC, and $L = 1$ m. Then frequency is $f = 1.2$ THz, bunch spacing $\Delta T = 207$ fs; the loss factor $\kappa_f = 180$ kV/(pC m), and chirp at the second bunch $h = 0.8$ MeV/fs

Uniform Distribution

- As long as the bunch lengths are small compared to the dechirper wavelength, using a Gaussian or uniform distribution results in similar results



Sketch of induced voltage in two-bunch configuration, comparing Gaussian bunches (red dashes) with bunches of uniform distribution (blue). The bunch positions and lengths are indicated by the flat rectangles, with their heads to the left.

Wake Calculation

- The bunch wake is given by

$$W_\lambda(s) = -\int_0^\infty w(s')\lambda(s-s')\,ds'$$

with $w(s)$ the point charge wake of the dechirper, *i.e.* $w(s) = 2H(s)\kappa\cos ks$, and $\lambda(s)$ the bunch (longitudinal) distribution. Here $\lambda(s) = 1/\ell$ (ℓ is full bunch length) for $|s| < \ell/2$, and 0 otherwise

- Performing the integration we find that

$$\begin{aligned} W_\lambda(s) &= 0, && \text{for } s < -\ell/2 \\ &= -\frac{2\kappa}{k\ell}\sin(ks + k\ell/2), && \text{for } -\ell/2 < s < \ell/2 \\ &= -\frac{4\kappa}{k\ell}\sin(k\ell/2)\cos ks, && \text{for } s > \ell/2 \end{aligned}$$

Wake Calculation Cont'd

- The bunch is short, and $\sin k\ell \approx k\ell$, so the chirp over the bunch is negative and nearly constant. Approximately, the chirp of the self-field is $h = -eQk\kappa L$, for either the leading or trailing bunch, Q_1 or Q_2, with L the dechirper length
- The chirp at the second bunch, due to the driving bunch at $\lambda/4$ ahead, is approximately $h = +2eQ_1 k\kappa L$
- The net chirp at the trailing bunch is then $h = 2ek\kappa L(Q_1 - \frac{1}{2}Q_2)$

With the higher frequency example, with $Q_2 = 100$ pC and $\ell_2 = 10$ μm ($I = 3000$ A), we obtain $h = 0.81$ MeV/fs without the self-fields, and $h = 0.67$ MeV/fs when they are included

Average energy loss of second bunch due to dechirper wake is $-\Delta\mathcal{E} = eQ_2\kappa L = 18$ MeV

Flat Geometry

- The above formulas for the longitudinal wake of a flat dechirper are only valid for a short distance behind the driving particle. Over the longer distance–the longer time scale–field will leak out the sides and the wake amplitude will decrease

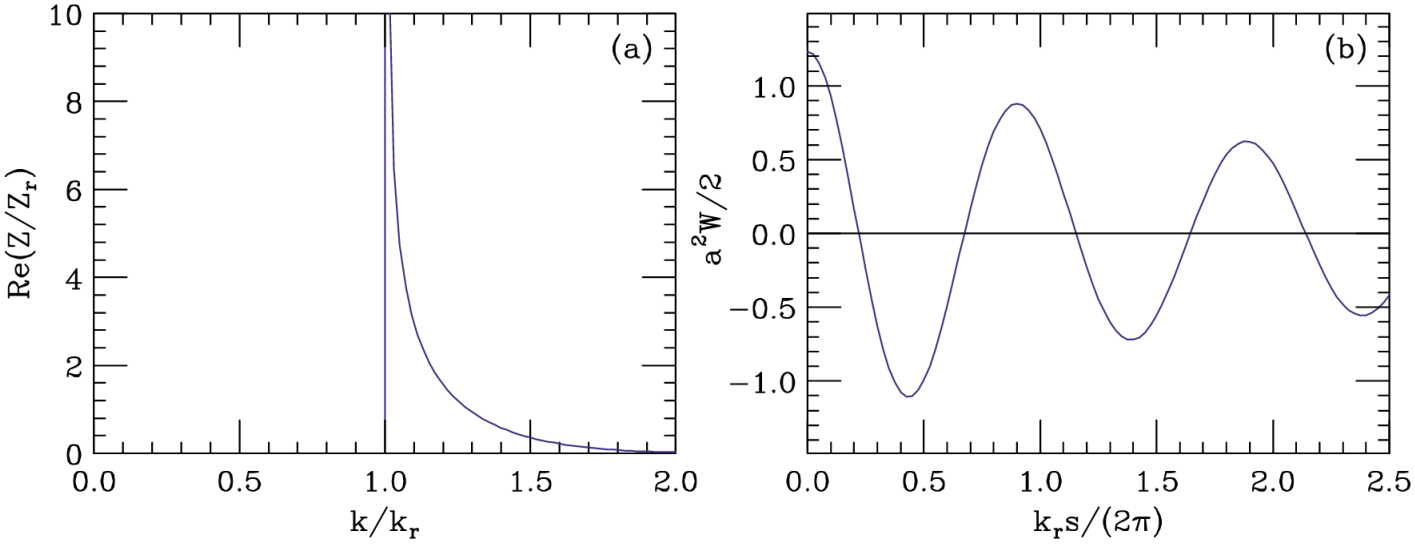


FIG. 3. (Color) For the case of two corrugated plates ($w \to \infty$): Re(Z) (a) and the wake (b), with $k_r = \sqrt{p/(a\delta g)}$ and $Z_r = \pi/(a^2 k_r c)$.

Result of perturbation calculations for the longitudinal impedance/wake excited between two corrugated, parallel plates. [K. Bane, G. Stupakov, PRST-AB 6, 024401 (2003)]

Transverse Wake: Self-Wake

• Consider a round dechirper, with a the iris radius. The dipole wakefield $w_x(s) \approx 2H(s)\kappa_{yr} \sin k_r s$, with kick factor $\kappa_{yr} = Z_0 c/(\pi a^4 k_r)$ and (the same) wave number $k_{yr} = \sqrt{2p/(g\delta a)}$

• With the second bunch behind by $\Delta T = \frac{1}{4}\lambda/c$, it will be at the peak of the dipole wake

• In a rectangular structure with the beam near the symmetry axis, the dominant transverse terms are the dipole and quad terms. For example, in a flat structure (oriented horizontally), the transverse wake (and also impedance) can be written in the form

$$w_y(s) = y_1 w_d(s) + y_2 w_q(s) \ , \quad w_x(s) = (x_1 - x_2) w_q(s)$$

where subscripts d and q stand for dipole and quad functions, *1* and *2* designate the leading and trailing particle, and s is the longitudinal separation of the particles

Transverse Wake: Self-Wake Cont'd

• There are only two functions. The dipole wake kicks the centroids of slices in a bunch all differently, and the quad wake defocuses/focuses the slices differently

• For the flat dechirper $w_d(s) = w_q(s)$, $k_y = k_{yr}/\sqrt{2}$ (I think), and $\kappa_{yd} = (\frac{\pi^4}{128}) \cdot Z_0 c/(\pi a^4 k_y)$. (But as we saw for the longitudinal case, there is a slight frequency spread.)

• For a short bunch of charge Q_2, with uniform λ of length ℓ_2, the emittance growth due to the dechirper is

$$\left(\frac{\epsilon_y}{\epsilon_{y0}}\right) = \left[1 + \left(\frac{\pi^3}{384\sqrt{5}} \frac{Z_0 c}{a^4} \frac{eQ_2 \beta_y L \ell_2}{E}\right)^2 \left(1 + 4\frac{y_2^2}{\sigma_y^2}\right)\right]^{1/2}$$

[K. Bane and G. Stupakov, NIM A 820 (2016) 156-163]

In the rightmost parentheses: The first term is due to the quad wake; it can be partially compensated by following a vertically orientated dechiper by a horizontal one. The second term is due to the dipole wake; it depends on the beam offset in the dechirper, y_2

Transverse Wake: Self-Wake Cont'd

- Consider again: $a = 0.25$ mm, $p = 25$ μm, $g = 6$ μm, $\delta = 25$ μm, $L = 1$ m, but half is vertical and half is horizontal. Then frequency is $f = 1.2$ THz, bunch spacing $\Delta T = 206$ fs

- $Q_1 = 300$ pC, $Q_2 = 100$ pC, $\ell_2 = 10$ μm; $\epsilon_{yn} = 0.5$ μm, $\beta_y = 20$ m, $E = 4$ GeV; $\sigma_y = 36$ μm.

The collection of parameters becomes

$$\zeta = \left(\frac{\pi^3}{384\sqrt{5}} \frac{Z_0 c}{a^4} \frac{eQ_2 \beta_y L \ell_2/2}{E} \right) = 2.6$$

The quad wake will largely be compensated by the different orientation of the two halves. The dipole wake effect, however, requires that the beam be steered to $|y_2| \lesssim 0.1\sigma_y$ to keep the emittance growth to 10%; this condition can be relaxed if ζ can be reduced by *e.g.* reducing β_y

Transverse Wake Effect of Leading Bunch on Trailing One: Injection Jitter

- Consider the effect of an injection jitter: a time-dependent variation of Bunch 1's transverse position (in the dechirper) so fast that Bunch 2's final orbit cannot be corrected

- Let each bunch be represented by a point particle. The kick experienced at Bunch 2 due to the offset in the dechirper of Bunch 1 is

$$\Delta y'_{12} = \frac{2eQ_1\kappa_{yd}Ly_1/2}{E} \ ,$$

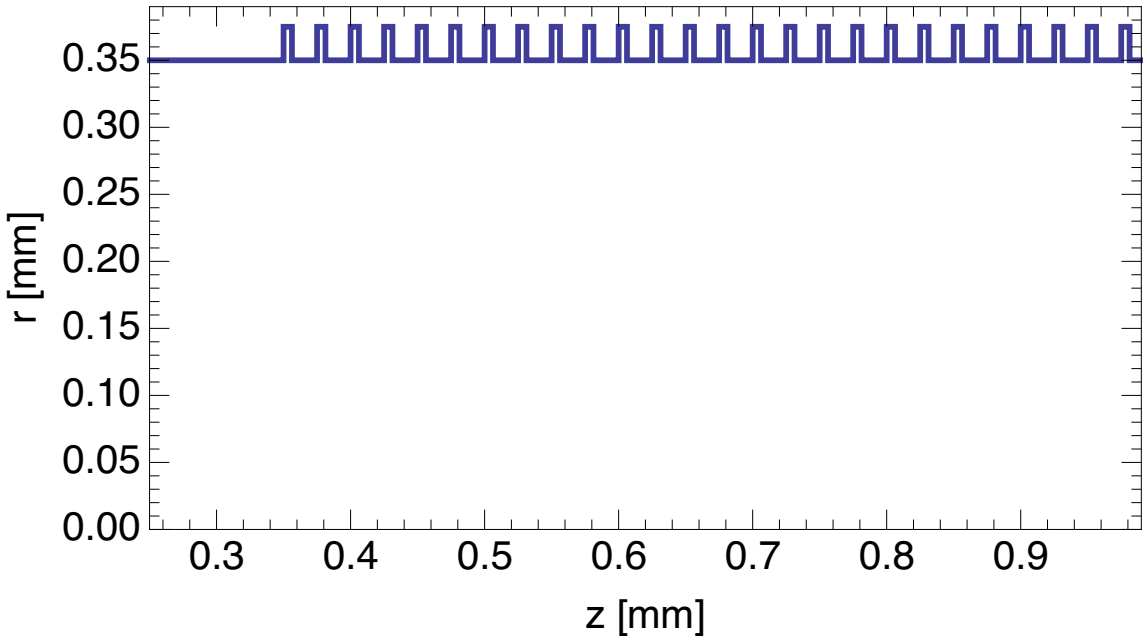
since Bunch 2 is 90° behind Bunch 1, in terms of the wake oscillation; *i.e.* on the wake peak. The relative emittance growth of Bunch 2 with respect to the axis is $\delta\epsilon_y \approx \frac{1}{2}(\Delta y'_{12}/\sigma_{y'})^2$, if the growth is not large.

For our example $\kappa_{yd} = 0.3$ MV/(pC*mm*m) and to keep the projected emittance growth to 20% we need to keep $y_1 < 55$ nm, or $y_1/\sigma_y < 0.0015$.

Echo simulations

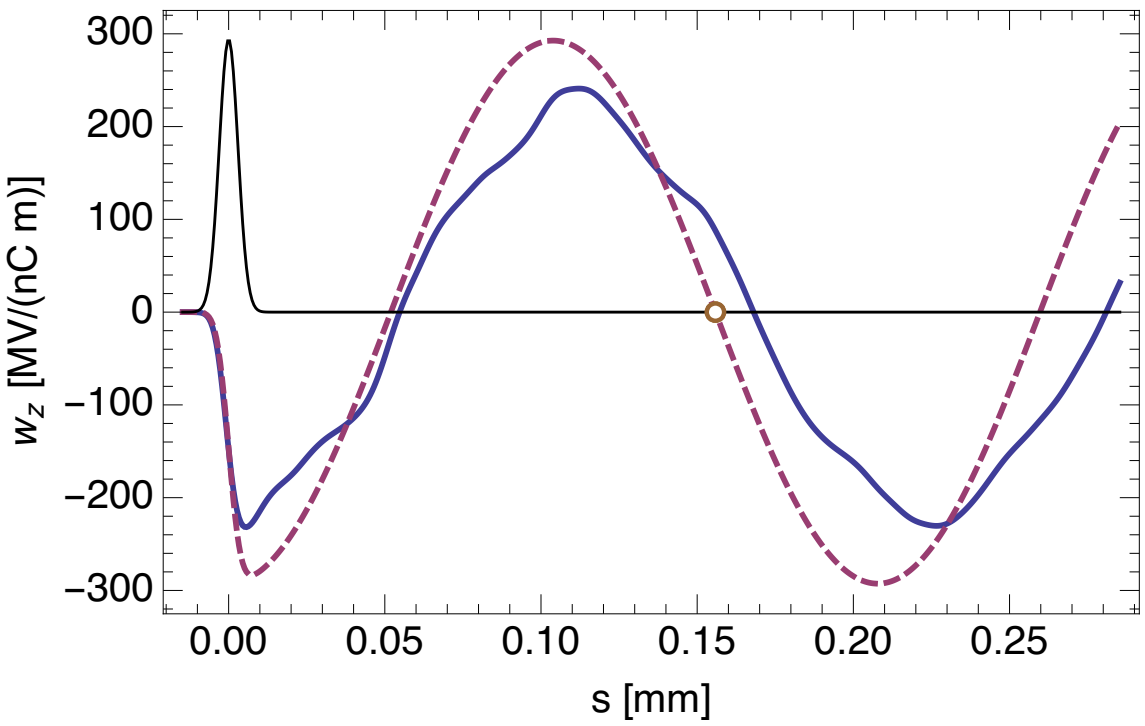
• I. Zagorodnov's cylindrically-symmetric wakefield solver, *Echo*, was used to test the accuracy of the analytical formulas for the round case. The driving bunch is Gaussian with $\sigma_z = 3$ μm

We don't need to solve for the wakes for the entire $L = 1$ m length. Sufficient to reach steady-state is for number of cells $N_c > a^2/(2\sigma_z p) = 800$, ($a = 350$ μm, $p = 25$ μm). I choose $N_c = 2000$ for the simulations



Beginning of structure geometry used in simulations

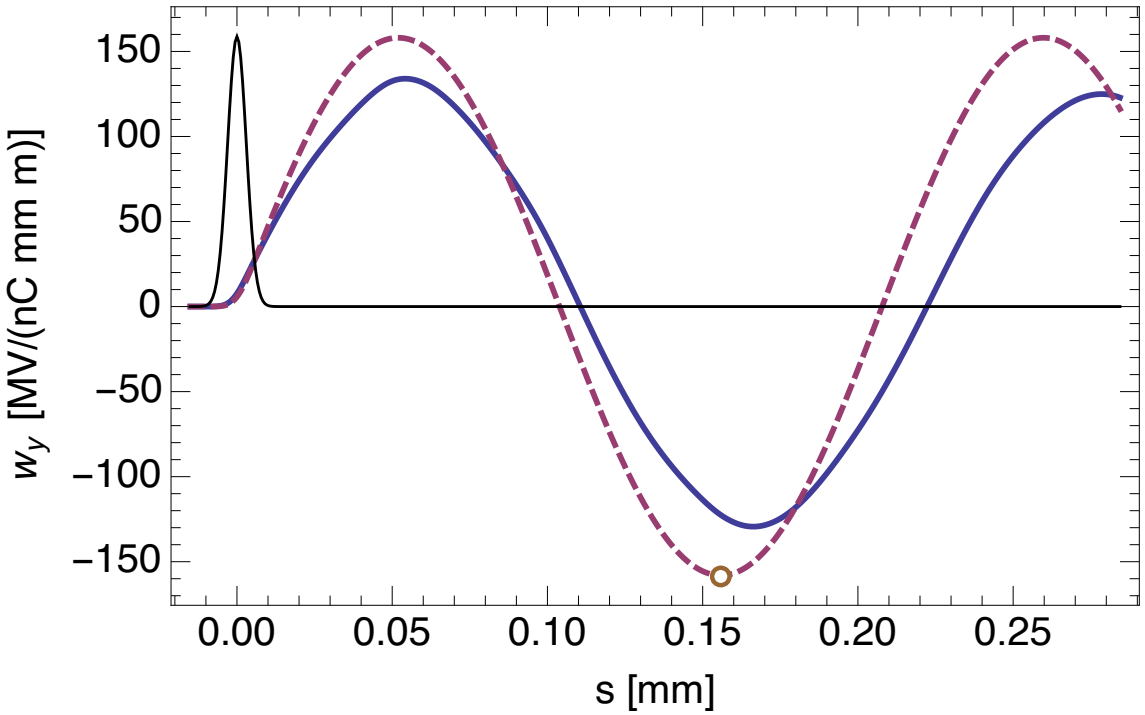
Longitudinal Wake for Round Structure



Longitudinal wake obtained by Echo (blue); the analytical approximation (red dashes), with the circle giving the second bunch location; the driving bunch shape is given in black, with the head to the left

- The calculation was rather quick

Dipole Wake for Round Structure



Dipole wake obtained by Echo (blue); the analytical approximation (red dashes), with the circle giving the second bunch location; the driving bunch shape is given in black, with the head to the left

- The calculation took 6–8 hours on my desktop

Leading Bunch to Trailing One: Injection Jitter Cont'd

- The analytical formulas for the round case are shown to be reasonably accurate
- The same exercise can be performed for the flat geometry using e.g. *Echo3D*, though it is not clear how easy it is to do the calculation we need

Three Cases

Table: Three cases that yield the same chirp, $h = 0.8$ MeV/fs, but have different jitter tolerances: *i.e.* the transverse jitter of Bunch 1 for 20% emittance growth of Bunch 2. Other parameters are $Q_1 = 300$ pC, $Q_2 = 100$ pC, normalized emittance $\epsilon_{yn} = 0.5$ μm, energy $E = 4$ GeV, corrugation period $p = 25$ μm, gap $g = 6.3$ μm, and structure length $L = 1$ m.

Parameter	A	B	C	Unit
Half aperture, a	0.25	0.33	0.33	mm
Corrugation depth, δ	25	6.3	6.3	μm
External focusing, β_y	20	20	2	m
Jitter tolerance, y_1	55	285	905	nm
Optimal bunch spacing, ΔT	205	120	120	fs

• Note: Decreasing β_y increases the jitter tolerance since relative emittance growth $\delta\epsilon_y \sim \Delta y'_{12}/\sigma_{y'}$, and $\sigma_{y'} \sim \beta_y^{-1/2}$

Dielectric-Lined, Mettalic Pipe

• A dielectric layer inside a metallic pipe can also act as a dechirper, with the main change in the formulas being $p/g \to \epsilon_m/(\epsilon_m - 1)$, with ϵ_m the dielectric constant, so that (in the round case)

$$k = \sqrt{\frac{2\epsilon_m}{(\epsilon_m - 1)a\delta_m}}$$

where δ_m is the layer thickness (A. Mosnier, A. Novokhatski, Proc. of PAC97, pp. 1661–3). In principle this can be tested with *Echo*

With $\epsilon_m = 2$ the formula is the same as for a corrugated pipe with $g = p/2$ and corrugation depth $\delta = \delta_m$; for ϵ_m large compared to 1 it becomes smaller by $1/\sqrt{2}$

At Argonne's Dielectric Wakefield Accelerator Facility (AWA) they have experimented with 2-mm thick Alumina ($\epsilon_m = 9.8$) and Frosterite ($\epsilon_m = 6.6$). We would need a hardy, lossless material that can be made uniform for a thickness $\delta_m \sim 25$ μm

Next Steps

- To explore the two-bunch plus dechirper idea further, a 3D simulation should be performed for the corrugated structure in flat geometry, for both longitudinal and transverse wakes. I think the main frequencies in the two cases will again be about the same, but now there will also be some frequency spread. In principle, *Echo3D* can be used for the simulations, though to date, for such structures, it has primarily been used to study the very short-range wakes.