LCLS-II Technical Note

Compensation of emittance growth from octupoles in BC1 and BC2 chicanes

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1 Abstract

Studies [1] showed that an octupole inserted in a bunch compressor chicane of a free electron laser can be effective for improving the bunch longitudinal phase space and removing the bunch current horns. Recently, this method has been studied in application to the LCLS-II [2], [3]. One of the issues of the octupole in a bunch compressor chicane is emittance growth due to the chicane dispersion and beam energy spread, which can be significant. In this paper, we propose a scheme to compensate this effect and analytically derive the compensation conditions. The analytic result assumes an ideal optics model and includes various approximations.

2 Compensating scheme

The LCLS-II bunch compressor system includes the BC1 and BC2 chicanes located within the SC linac. For this study, let us consider part of the LCLS-II consisting of the BC1, the L2 linac, and the BC2. Assume an octupole OBC2 is inserted in the BC2 chicane, where dispersion is sufficiently large, for the beam shaping. Due to the dispersion, the octupole would create an unwanted growth of horizontal emittance [4], [5]:

$$\frac{\epsilon}{\epsilon_0} = \sqrt{1 + \frac{5}{12}(K_3 L)^2 (\eta\sigma_\delta)^6 \frac{\beta}{\epsilon_0}} \tag{1}$$

where $K_3 = \frac{B'''}{B\varrho}$ is the octupole K-value, L is the octupole length, η and β are horizontal dispersion and horizontal beta function at the octupole, σ_δ is Gaussian sigma of relative energy spread, and $\epsilon_0 = \frac{\gamma\epsilon_0}{\gamma}$ is un-normalized initial emittance. For the parameters discussed in this paper, the emittance growth would be a factor of five or more which is unacceptable.

From Eq. (1), one can see that this emittance growth is a chromatic effect. It is caused by the octupole non-linear field distorting the horizontal orbits (X and X′) of the off-energy electrons, which ultimately cause an increase of the beam horizontal phase space. To compensate the emittance growth from the OBC2 octupole, we need to cancel the off-energy orbit distortions which it creates. To do so, we propose to insert a second octupole OBC1 in the BC1 chicane, upstream of the BC2 and L2. In this scheme, the OBC1 creates a similar orbit distortion, but opposite to the one made by the OBC2, so the two distortions cancel each other after OBC2. A schematic of such orbit cancellation for an off-energy electron is shown in Figure 1. To be effective, this scheme must work for all the electrons within the bunch energy spread.

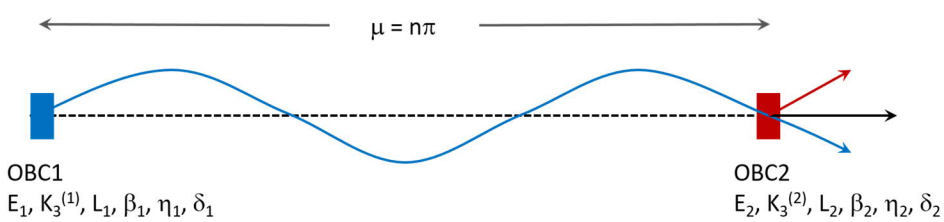

Figure 1: Schematic of horizontal orbit distortion of off-energy electron created by OBC1 (blue) which is then cancelled by opposite orbit distortion from OBC2 (red), where E, K_3, L, β, η, δ are the beam energy, octupole K-value, octupole length, horizontal beta function, first order dispersion, and relative energy offset of the electron, respectively, at the OBC1 and OBC2 locations. Dash line is undistorted orbit (octupoles off).

In Figure 1, the OBC1 makes an horizontal angular kick on the orbit of electron with relative energy offset δ_1; this results in orbit distortion shown in blue color which propagates downstream and oscillates as $\sin\mu$, where μ is horizontal phase advance. The OBC2 octupole also makes a kick on the same electron which would also result in an orbit distortion propagating downstream. In order to cancel the sum of these orbits, we need to satisfy two requirements. The first one is that the two orbit waves are in the same betatron phase, i.e. the OBC1 and OBC2 are separated by $\mu = n\pi$. This makes both waves to cross zero (X = 0) at the OBC2. The second requirement is that the two orbits at the OBC2 have equal and opposite angles X'. Since the two orbits add linearly, their sum vanishes after the OBC2, thus restoring the original undisturbed electron orbit and, hence, cancelling the effect on emittance.

The OBC2 octupole strength is primarily determined from the beam shaping optimization, hence the available free parameter is the strength of the OBC1 octupole which needs to be set to satisfy the above cancellation conditions. Since the emittance growth is a cumulative effect on all the electrons in the beam, the proposed scheme should work for all the electrons for the same octupole strengths. Below we perform analytical calculations with the goal to derive the octupole compensating strengths.

Apart from the octupole non-linear effect, other effects in this analysis will be limited to first order. We consider the ideal optics without errors; also wakefield effect in the L2 linac is not included. Due to these approximations and limitations, the resulting compensation may be approximate. It will be compared with numeric optimization using tracking simulations.

3 Octupole effect

As a first step, we evaluate the effect of an octupole on orbit of an electron with relative energy offset δ. The octupole field seen by electron has this form:

$$B_y = \frac{B'''}{6}(x^3 - 3xy^2) \tag{2}$$

$$B_x = \frac{B'''}{6}(3x^2y - y^3) \tag{3}$$

where x and y are the electron horizontal and vertical coordinates relative to the octupole center (assuming the octupole is aligned on the beam nominal trajectory). Taking into account the first-order horizontal dispersion (η) in the BC1,2 chicanes, we re-write the electron coordinates as $x = x_\beta + \eta\delta$ and $y = y_\beta$, where x_β and y_β represent the betatron part of the coordinates after subtracting the first order dispersion and neglecting the higher order dispersion. Note that the large BC1,2 dispersion dominates the electron horizontal positions, i.e. $|\eta\delta| \gg |x_\beta|$. With the above substitution, the octupole field becomes:

$$B_y = \frac{B'''}{6}\left(x_\beta^3 - 3x_\beta y_\beta^2\right) + \frac{B'''\eta\delta}{2}\left(x_\beta^2 - y_\beta^2\right) + \frac{B'''\eta^2\delta^2}{2}x_\beta + \frac{B'''\eta^3\delta^3}{6} \tag{4}$$

$$B_x = \frac{B'''}{6}\left(3x_\beta^2 y_\beta - y_\beta^3\right) + \frac{B'''\eta\delta}{2}\left(2x_\beta y_\beta\right) + \frac{B'''\eta^2\delta^2}{2}y_\beta \tag{5}$$

The first term in both equations is the purely geometric octupole field (independent of δ). The other terms (lower order) sometimes are called feed-down terms which result from the octupole field expansion relative to the dispersive orbit ($\eta\delta$). The second term is chromatic (dependent on δ) sextupole field, the third term is chromatic quadrupole field, and the last term in the B_y equation is chromatic dipole field. The chromatic quadrupole and sextupole produce δ-dependent linear focusing and 2nd order effects on the off-energy electrons; and the octupole creates the 3rd order geometric effects independent of δ. The B_x and B_y fields of

the quadrupole, sextupole and octupole act in both horizontal and vertical planes, and their effect depends on the electron x_β and y_β coordinates. The last term of the B_y field is the chromatic dipole which creates the 3rd order horizontal dispersion, i.e. orbit proportional to δ^3. Unlike the other terms, it makes the effect only in the horizontal plane which is also independent of x_β and y_β. Since $|\eta\delta| \gg |x_\beta|$, it is clear that the chromatic dipole field is the dominant term in Eq. (4), and therefore the 3rd order horizontal dispersion produced by this field is the primary source of the horizontal emittance growth. This is also evident when examining Eq. (1). Therefore, to compensate the emittance growth, our goal is to cancel this 3rd order dispersive orbit.

The chromatic dipole field in Eq. (4) changes the electron horizontal orbit angle at the octupole by this amount:

$$\theta = -\frac{B_y L}{B\rho} = -\frac{1}{6} K_3 L \eta^3 \delta^3 \tag{6}$$

where $B_y = \frac{B'''\eta^3\delta^3}{6}$ is the last term in Eq. (4). Since $\theta \sim \delta^3$, it is the 3rd order angular dispersion; it gives rise to the 3rd order dispersive orbit propagating downstream as schematically shown in Figure 1. Our goal is to cancel the propagation of such orbit from the OBC2 by creating a compensating orbit using a second octupole (OBC1) in the BC1.

4 Compensation of the 3rd order orbit

From Eq. (6), the 3rd order horizontal orbit angles created by the OBC1 and OBC2 octupoles on the same electron with the parameters defined in Figure 1 are:

$$\theta_{11} = -\frac{1}{6} K_3^{(1)} L_1 \eta_1^3 \delta_1^3 , \qquad \theta_{22} = -\frac{1}{6} K_3^{(2)} L_2 \eta_2^3 \delta_2^3 \tag{7}$$

where θ_{11} is the angle created by OBC1 at its location in BC1, and θ_{22} is the angle from OBC2 at its location in BC2. Note that the energy offsets δ_1 and δ_2 in the Eq. (7) are for the same electron; they are different in the BC1 and BC2 due to the acceleration in L2.

To determine the compensating octupole strengths, we need to propagate the OBC1 orbit downstream and determine its angle θ_{12} at the OBC2 location. For the cancellation of the OBC1 and OBC2 orbits, the sum of their angles at the OBC2 location must vanish, assuming the horizontal phase advance between the octupoles is $\mu = n\pi$, i.e.

$$\theta_{12} + \theta_{22} = 0 \tag{8}$$

We apply transfer matrix to propagate the OBC1 orbit. Small focusing effects due to δ-dependence are neglected, i.e. the nominal beta functions are used. Based on the general matrix form, the OBC1 orbit propagated to a downstream point will have amplitude X and angle X′:

$$X = R_{12}\theta_{11} = \theta_{11}\sqrt{\beta_1\beta}\sqrt{\frac{E_1}{E}}\sin\mu \tag{9}$$

$$X' = R_{22}\theta_{11} = \theta_{11}\sqrt{\frac{\beta_1}{\beta}}\sqrt{\frac{E_1}{E}}(\cos\mu - \alpha\sin\mu) \tag{10}$$

where the Twiss functions β, α, μ, and the energy E without an index are at the observation point. In the

above equations, we take into account the fact that the normalized emittance during acceleration stays the same, so the X and X′ sizes shrink as $\frac{1}{\sqrt{E}}$. Applying the $\mu = n\pi$ phase advance between the octupoles, the X and X′ of the OBC1 orbit at the OBC2 location become:

$$X = 0, \quad X' = \theta_{12} = \theta_{11}\sqrt{\frac{\beta_1 E_1}{\beta_2 E_2}}(-1)^n = -\frac{1}{6}K_3^{(1)}L_1\eta_1^3\delta_1^3\sqrt{\frac{\beta_1 E_1}{\beta_2 E_2}}(-1)^n \tag{11}$$

where we substitute the θ_{11} with its expression in Eq. (7), and the β_2 and E_2 are the beta function and energy at the OBC2. By our earlier designation, θ_{12} is the OBC1 orbit angle at the OBC2 location. Finally, substituting the θ_{12} and θ_{22} from Eq. (11) and (7) into Eq. (8) yields:

$$\frac{K_3^{(1)}L_1}{K_3^{(2)}L_2} = (-1)^{n+1}\sqrt{\frac{\beta_2 E_2}{\beta_1 E_1}}\left(\frac{\eta_2}{\eta_1}\right)^3\left(\frac{\delta_2}{\delta_1}\right)^3 \tag{12}$$

The Eq. (12) defines the strength ratio of the two octupoles for compensating the 3rd order orbits they create. For emittance compensation, this condition needs to be valid for all the electrons with different δ. Since the beta function and dispersion in Eq. (12) are given by the optics, then we only need to determine the ratio of $\frac{\delta_2}{\delta_1}$ and whether it is the same for all the electrons; this will be investigated below. In a special case of no acceleration between the octupoles, where $\delta_1 = \delta_2$ and $E_1 = E_2$, the octupole strength ratio depends only on beta function and dispersion.

Note that the dispersion η in Eq. (12) is proportional to the angle θ_B of the chicane bend and, hence, to the $\sqrt{R_{56}}$ of the chicane. So, one could, in principle, replace the $\frac{\eta_2}{\eta_1}$ with the ratio of the latter quantities, however, with the additional factor depending on the BC1 and BC2 parameters and the octupole locations. For example, if the octupoles are placed in the middle drift of each chicane, then to first order $\frac{\eta_2}{\eta_1} = \left(\frac{\theta_{B2}}{\theta_{B1}}\right)\left(\frac{L_{B2}+L_{D2}}{L_{B1}+L_{D1}}\right)$, where L_B and L_D are the lengths of the chicane bend and the drift between the outer and the inner bends, respectively. If at least one octupole is placed outside of the middle drift, then the multiplication factor in the latter ratio would have to be changed appropriately. For general purpose, we will keep using the dispersion ratio in Eq. (12), since it is compatible with any octupole location (inside or outside of the middle drift).

5 Transformation of δ

In order to evaluate the Eq. (12), we need to determine the transformation of δ from BC1 to BC2. This can be done by obtaining the 6×6 transfer matrix between the octupoles. This matrix is a product of individual matrices of drifts, bends, quads and accelerating cavities in this area. Obtaining the full 6×6 matrix may be complicated, however, it is not necessary for our purpose. To simplify our task we will take into account the properties of the individual matrices and the fact that only the R_{65} and R_{66} terms of the final matrix are needed to determine δ.

We divide the area between the octupoles into three regions: 1) from BC1 octupole to the beginning of L2, 2) the L2 linac, and 3) from the L2 to BC2 octupole. The 6×6 matrix of the chicane regions 1) and 3) has the following form:

$$R^{(1,2)} = \begin{vmatrix} A^{(1,2)} & 0 & D^{(1,2)} \\ 0 & B^{(1,2)} & 0 \\ E^{(1,2)} & 0 & C^{(1,2)} \end{vmatrix} \tag{13}$$

Inside the above matrix each term is a 2×2 matrix, the indexes 1,2 are for the BC1, BC2 regions, respectively, the A, B and C matrices relate to horizontal, vertical and longitudinal transformation, respectively, and D and E relate to dispersion. Specifically, of our interest are the matrices:

$$C^{(1,2)} = \begin{vmatrix} 1 & R_{56}^{(1,2)} \\ 0 & 1 \end{vmatrix}, \quad D^{(1,2)} = \begin{vmatrix} 0 & R_{16}^{(1,2)} \\ 0 & R_{26}^{(1,2)} \end{vmatrix}, \quad E^{(1,2)} = \begin{vmatrix} R_{51}^{(1,2)} & R_{52}^{(1,2)} \\ 0 & 0 \end{vmatrix} \tag{14}$$

where $R_{56}^{(1,2)}$ is the 56-term for the part of the chicane between the octupole and L2.

Using similar representation, the 6×6 accelerator cavity matrix can be expressed through 2×2 matrices as follows:

$$R^L = \begin{vmatrix} F & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & H \end{vmatrix} \tag{15}$$

The explicit form of this matrix is given in Equation 25 in the Appendix-1 at the end of this note, taken from TRANSPORT manual by K. L. Brown [6]. In the TRANSPORT, as described in the Appendix-1, the RF phase lag φ is defined to be positive if the bunch is behind the crest. In this note, however, we use the definition where the RF phase is negative when the bunch is behind the crest. For this reason, we change the sign of the R_{65} expression as compared to the expression in the Appendix-1; the other terms do not change. It is also important to mention that we use the same definition of "z" coordinate as in the TRANSPORT, namely it can be thought of as path length difference relative to the nominal particle, or the electron "z" position in the bunch, where the head is at z < 0.

The L2 linac consists of accelerating cavities, quads and drifts. The drifts and quads have the matrix form similar to Eq. (15), but with the longitudinal 2×2 matrix being almost an identity matrix (we neglect a small 56-term $= -\frac{L}{\gamma^2}$). With this approximation, the quads and drifts do not affect the longitudinal matrix, so we completely ignore them here, using only the accelerator section matrix in Eq. (15). Therefore, to obtain the longitudinal part of the complete OBC1-L2-OBC2 matrix, it is sufficient to calculate this matrix:

$$R = R^{(2)} R^L R^{(1)} \tag{16}$$

Substituting the Eq. (13) and (15) into the (16) gives the following longitudinal matrix:

$$\begin{vmatrix} R_{55} & R_{56} \\ R_{65} & R_{66} \end{vmatrix} = C^{(2)} H C^{(1)} + E^{(2)} F D^{(1)} \tag{17}$$

where the terms on the right hand side are products of 2×2 matrices. From Eq. (14) and Equation 25 one can see that only the $C^{(2)} H C^{(1)}$ term in Eq. (17) contributes to the R_{65} and R_{66} which are:

$$R_{65} = R_{65}^L, \quad R_{66} = R_{66}^L + R_{56}^{(1)} R_{65}^L \tag{18}$$

Here, $R_{56}^{(1)}$ is the 56-term of the matrix from OBC1 to L2, and the R_{65}^L, R_{66}^L terms are given in Equation 25, except that the sign of the R_{65}^L term is reversed since we use the opposite sign of the RF phase φ as compared to the Equation 25 definition:

$$R_{65}^L = -\frac{2\pi f}{c}\left(1 - \frac{E_1}{E_2}\right)\tan\varphi, \quad R_{66}^L = \frac{E_1}{E_2} \tag{19}$$

where $f = 1.3\ GHz$ is the RF-frequency of LCLS-II linac, c is the speed of light, and the RF phase φ is negative for the bunch behind the crest. Inserting the Eq. (19) into (18) yields:

$$R_{65} = -\frac{2\pi f}{c}\left(1-\frac{E_1}{E_2}\right)\tan\varphi\ , \quad R_{66} = \frac{E_1}{E_2} - R_{56}^{(1)}\frac{2\pi f}{c}\left(1-\frac{E_1}{E_2}\right)\tan\varphi \tag{20}$$

With these matrix terms, we can obtain the transformation of δ from BC1 to BC2:

$$\delta_2 = R_{65}z_1 + R_{66}\delta_1 \tag{21}$$

Due to the beam chirp in the BC1, the δ_1 and z_1 are correlated. This dependence is mostly linear, so in order to solve the equation, we make a simplification and neglect the small non-linearity assuming that the electron longitudinal position z_1 at the OBC1 octupole is a linear function of δ_1:

$$z_1 = \frac{\delta_1}{h_1} \tag{22}$$

where $h_1 > 0$ and the bunch head is at $z_1 < 0$ and $\delta_1 < 0$.

Then, from Eq. (20) - (22):

$$\frac{\delta_2}{\delta_1} = \frac{R_{65}}{h_1} + R_{66} = \frac{E_1}{E_2} - \left(\frac{1}{h_1} + R_{56}^{(1)}\right)\frac{2\pi f}{c}\left(1-\frac{E_1}{E_2}\right)\tan\varphi \tag{23}$$

One can see that within this model the ratio $\frac{\delta_2}{\delta_1}$ is independent of δ, hence the compensating condition in Eq. (12) works for all the electrons within the energy spread. Inserting this equation into the Eq. (12) yields the final formula for the compensating octupole strengths:

$$\frac{K_3^{(1)}L_1}{K_3^{(2)}L_2} = (-1)^{n+1}\sqrt{\frac{\beta_2 E_2}{\beta_1 E_1}}\left(\frac{\eta_2}{\eta_1}\right)^3\left[\frac{E_1}{E_2} - \left(\frac{1}{h_1} + R_{56}^{(1)}\right)\frac{2\pi f}{c}\left(1-\frac{E_1}{E_2}\right)\tan\varphi\right]^3 \tag{24}$$

Since this condition defines the ratio of the octupole strengths, the emittance compensation should work even if the octupole strengths are varied, provided that their ratio is kept the same as in Eq. (24).

6 Model limitations and discussion

The presented analytic derivation of the BC1 and BC2 octupole emittance compensation is performed within certain approximations and limitations. Specifically, we considered only linear optics effects except the octupole non-linear kick, did not include dependence of optics functions on δ, and neglected non-linearity of the beam chirp. Also, the effect of wakefield in the L2 linac is not included. Tracking simulations with the wakefield turned on and off show that this effect does not significantly affect the compensation condition for this beamline [7].

Another consideration is that the octupole orbit distortions are compensated only after the BC2 octupole. Before the OBC2 the orbit distortions generated by the OBC1 may still be large. If the OBC1 is too strong, the large electron orbits may cause unacceptable beam loss in this area. Therefore, it is important to keep the OBC1 strength sufficiently low. Minimization of the beam loss was included in the beam shaping optimization [2]. Losses of <1% are achieved in the simulations, which should be acceptable at this low beam energy. Note that according to Eq. (24), the $K_3^{(1)}$ of the OBC1 could be reduced as $\frac{1}{\sqrt{\beta_1}}$ by increasing the octupole beta function β_1, however this would not affect the beam loss since the orbit distortions are proportional to $K_3^{(1)}L_1\sqrt{\beta_1}$ which stays the same with such manipulation.

Since an octupole most strongly deflects electrons with large amplitude, it may affect the beam halo and the beam dark current. Some of the halo electrons may be lost as discussed above, but some electrons may be transferred into the halo, for example electrons in the energy tails may be transferred into the transverse tails of the halo due to the large dispersion at the octupole. Particle tracking from BC1 to BC2 of a test halo beam with large normalized emittance of 100 μm-rad, 2% Gaussian sigma of energy spread, and 2×10^5 electrons was performed [7] to estimate the effect of the octupoles on beam loss in one of the beam shaping configurations [3]. In this test, most of the halo electrons are absorbed in the collimators between the BC1 and L2. When the octupoles are turned off, 96.9% of the test halo is lost on the collimators, while with the octupoles turned on, 97.6% of the halo is lost due to the BC1 octupole deflecting some of the electrons to larger amplitude. Since the halo beam is more collimated when the octupoles are on, the remaining halo has a factor of two less additional losses downstream of the collimators than the halo beam with the octupoles off. After the BC2, the test halo with the octupoles turned on has ~17% fewer electrons in the beam compared to the case without octupoles. In both cases, the halo beams have similar transverse size.

The octupole field required for the beam shaping and emittance compensation must be realistically achievable. Due to the large BC1 and BC2 dispersion, the aperture of the OBC1 and OBC2 must be at least 70 mm and 50 mm, respectively, as specified in the LCLS-II BC1 and BC2 Physics Requirements Documents. The octupole pole-tip field scales proportionally to the beam energy, the K_3 value, and the cube of the octupole aperture. For the OBC1 and OBC2 placed at center of the chicanes, the typical K_3L values in the optimization study [2], [3] are below 4000 m^{-3} and 400 m^{-3}, respectively. Assuming the octupole length is 0.2 m and the OBC1 and OBC2 apertures are 70 mm and 50 mm, respectively, the corresponding pole-tip field is 1.19 kG and 0.28 kG, which is well within the reach. If needed, the octupole could be made shorter or moved outside of the middle drift without exceeding the field limit.

So far we have discussed an ideal optics without errors. But unavoidable octupole alignment errors would also cause emittance growth. Similar to Eq. (4)-(5), an octupole offset creates the effects of feed-down sextupole, quadrupole and dipole on the electron trajectory leading to enlargement of the beam phase space. Octupole alignment tolerances must be set to limit the corresponding emittance growth to an acceptable level. Based on Eq. (26) in the Appendix-2 [5], the octupole alignment tolerances corresponding to the octupole strengths in the shaping study are approximately 160 and 240 μm for the BC1 and BC2 octupoles, corresponding to 10% of emittance growth per octupole. These tolerances were also estimated in tracking simulations [4] using elegant [8] and found to be in a similar range. Achieving such tolerances should not be a significant issue.

Finally, the beam shaping requires that the BC1 and BC2 octupole strengths are of the same sign. Based on Eq. (24), this can be achieved with the same bending polarity in the BC1 and BC2 and 180º phase advance between the octupoles ("n" is odd number in Eq. (24)), or the opposite BC1 and BC2 polarities and 360º phase advance ("n" is even number). The two chicanes, as designed, are of the opposite polarity and the design phase advance is almost exactly 3π, which is the wrong condition for the beam shaping. To satisfy the shaping condition, the phase advance could be increased to 4π. The latter is possible by increasing the FODO cell phase advance in the L2 linac from 30º to about 45º per cell. The required stronger quadrupoles in the L2 and in the matching sections are still well within their field range since they are designed for at least 4 GeV energy as compared to 1.6 GeV in BC2. The stronger quadrupole focusing also increases chromaticity. The impact of the chromaticity on the octupole compensation is due to the modest increase of the spread of the off-energy electron's phase advances between the octupoles (~ couple of degrees); this is not expected to be a significant effect.

7 Example

In this example, we compare the compensating octupole strengths based on the above theory with numeric optimization obtained in particle tracking simulation [3]. We use beam shaping configuration A5 from [3] where the octupole strengths are optimized in tracking using elegant. In this configuration, the octupoles are placed near the center of the chicanes, the phase advance between the octupoles is 3π (n = 3), the BC1 and BC2 chicane polarities are artificially made to be the same (η_1 and η_2 are of the same sign), and the other optics and beam parameters are listed in Table 1.

Table 1: Configuration A5 parameters

β_1, m	β_2, m	η_1, m	η_2, m	E_1, GeV	E_2, GeV	$R_{56}^{(1)}$, m	h_1, m^{-1}	φ, deg	f, Hz
10.96	60.18	0.275	0.494	0.25	1.6	-0.0275	18.24	-34.2	1.3×10^9

Note that the $R_{56}^{(1)}$ in this table is the 56-term of the matrix from the BC1 octupole to L2; it is half of the chicane total R_{56}.

The elegant optimization resulted in the octupole strength ratio of $\frac{K_3^{(1)}L_1}{K_3^{(2)}L_2} = 8.3$ which besides the beam shaping and emittance compensation includes other considerations such as minimization of beam loss. The simulations include the realistic beam distribution, the chirp non-linearity, and the non-linear optics effects. Evaluation of the Eq. (24) with the configuration A5 parameters yields the octupole strength ratio of 6.8. This is within 20% of the elegant result. Given the limitations of the analytic model, this result may be considered as a reasonable initial prediction of the octupole strength ratio and used as initial setting for numeric optimization.

8 Acknowledgements

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9 Appendix-1

Equation 25: Accelerator section matrix from TRANSPORT [6].

$$\begin{bmatrix} 1 & \left[L \frac{E_0}{\Delta E \cos\phi} \ln\left(1 + \frac{\Delta E \cos\phi}{E_0}\right)\right] & 0 & 0 & 0 & 0 \\ 0 & \frac{E_0}{E_0 + \Delta E \cos\phi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \left[L \frac{E_0}{\Delta E \cos\phi} \ln\left(1 + \frac{\Delta E \cos\phi}{E_0}\right)\right] & 0 & 0 \\ 0 & 0 & 0 & \frac{E_0}{E_0 + \Delta E \cos\phi} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \left(\frac{\Delta E \sin\phi}{E_0 + \Delta E \cos\phi}\right)\left(\frac{2\pi}{\lambda}\right) & \frac{E_0}{E_0 + \Delta E \cos\phi} \end{bmatrix}$$

Definitions:

L = effective length of accelerator sector.

E_0 = particle energy at start of sector.

ΔE = energy gain over sector length.

ϕ = phase lag of the reference particle behind the crest of the accelerating wave, i.e. if ϕ is positive then for some $\ell > 0$ the particles having this value are riding the crest of the wave; the units of ϕ are degrees.

λ = wavelength of accelerating wave; the units of λ are those of ℓ (normally cm).

10 Appendix-2

Analytic estimate of relative horizontal emittance growth due to octupole horizontal misalignment is obtained using [5]:

$$\frac{\Delta\epsilon_x}{\epsilon_{xo}} = \frac{3}{8}(K_3 L \Delta x_o)^2 \beta_{xo}^3 \epsilon_{xo}\left(1 + \frac{(\eta_{xo}\sigma_\delta)^2}{\beta_{xo}\epsilon_{xo}}\right)^2 \tag{26}$$

where ϵ_{x0} is un-normalized horizontal emittance, and Δx_0 is the octupole horizontal offset error. The other

parameters are defined elsewhere in the paper. Parameters with index 'o' are taken at the octupole location. The equation (26) is obtained, first, by writing down the octupole feed-down sextupole, quadrupole and dipole fields on the electron trajectory due to Δx_0 (similar to Eq. (4)-(5)), and then deriving emittance growth due to the feed-down sextupole as the dominant term [5]. In this derivation, the emittance growth is assumed to be small (<<1). It scales with the square of Δx_0 and $K_3 L$, and is greatly enhanced by the large dispersion since the dispersive beam size $\eta_{xo}\sigma_\delta$ is much larger than the betatron size $\sqrt{\beta_{xo}\epsilon_{xo}}$. In fact, for the BC1 and BC2 parameters one can simplify the expression in Eq. (26) by neglecting the term "1" in the brackets:

$$\frac{\Delta\epsilon_x}{\epsilon_{xo}} \approx \frac{3}{8}(K_3 L \Delta x_o)^2 \frac{\beta_{xo}}{\epsilon_{xo}}(\eta_{xo}\sigma_\delta)^4 \tag{27}$$

For the Δx_o tolerance estimate, we assume $\gamma\epsilon_{x0} = 0.5\,\mu m$, 10% of emittance growth, $K_3 L = 4000\,m^{-3}$ in BC1 and 400 m^{-3} in BC2, and Gaussian $\sigma_\delta = 1\%$ and 0.6% in the BC1 and BC2, respectively. The other parameters are listed in Table 1.

11 References

[1] T. K. Charles, D. M. Paganin, A. Latina, M. J. Boland and R. T. Dowd, "Current-horn suppression for reduced coherent-synchrotron-radiation-induced emittance growth in strong bunch compression," Phys.Rev.Accel.Beams, vol. 20, no. 3, 2017.

[2] N. Sudar, K. Bane, Y. Ding, Y. Nosochkov and Z. Zhang, "Non-linear compression with octupoles for current profile shaping," LCLS-II-TN-20-05, July 2020.

[3] Y. Ding, "Beam shaping update with octupoles," Study report (unpublished), Dec 4, 2019.

[4] N. Sudar, private communication.

[5] P. Emma, private communication.

[6] K. L. Brown and F. Rothacker, "Transport: A computer program for designing charged particle beam transport systems," SLAC-91, rev. 3, 1983.

[7] Y. Ding, private communication.

[8] M. Borland, "elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation," Advanced Photon Source LS-287, September 2000.