LCLS-II Technical Note

Scattering in LCLS-II and loss rates on the collimators

LCLSII-TN-15-14

5/1/2015

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There are four processes that we will consider: 1) elastic Coulomb scattering on the nucleus, 2) bremsstrahlung off the atom, 3) elastic scattering off the atomic electrons, and 4) Thomson scattering off thermal photons. Processes 1 and 2 will be the largest effect while 3 and 4 are less accurate corrections. Throughout we will assume $\gamma >> 1$ and $\beta \sim 1$. Finally, we discuss the expected partial pressures compared to the Nitrogen-equivalent pressure that most gauges measure.

1. <u>Coulomb scattering - Nucleus:</u>

The Born approximation with the Fermi-Thomas model for the atomic potential yields a differential cross section of [1,3]:

$$\frac{d\sigma}{d\Omega} \approx \left(\frac{2Zr_0}{\gamma}\right)^2 \frac{1}{\left(\theta^2 + \theta_{min}^2\right)^2}$$

Where Z is the atomic number, r_0 is the classical electron radius, q is the scattering angle and θ_{min} is a function of the atomic screening: $\theta_{min} \sim \alpha Z^{1/3}(mc/p) = Z^{1/3}/192\gamma$ with m the electron mass, p the momentum, and γ is the relativistic factor. The maximum scattering angle is given by the maximum momentum transfer due to the finite nuclear size and is $\theta_{max} \sim 274/A^{1/3}\,\gamma$ where A is the atomic number; in most cases, we can neglect θ_{max} as it is large compared to the angles of interest.

We are interested in the number of particles scattered to large amplitude so they hit collimators. The collimator jaws are assumed to have half apertures of ax and ay in the horizontal and vertical planes.

Now at some location along the beamline, the rate of scattering into an X collimator is:

$$\frac{dN}{cdt} = n_{gas} \int_{ax/|R_{12}Cos\phi|}^{\theta_{max}} \frac{d\sigma}{d\Omega} d\Omega$$

Where R12 is the transport element from the scattering location to the collimator, $\cos\phi$ is the azimuthal dependence on the scattering, and the absolute value appears because we assume two jaws located at ±ax. We will further assume that the scattering angles are large compared to q_{min} and we'll ignore q_{max}. In this case, the expression reduces to:

$$\frac{dN}{cdt} = 2\pi N_b n_{gas}(s) \left(\frac{Zr_0 R_{12}(s)}{ax\,\gamma(s)}\right)^2$$

where n_{gas} is the atomic gas density which, at 20°C, is roughly $3.2\text{x}10^{22}$ molecules/Torr/m^3 and N_b is the number of beam particles. This expression (or the equivalent for the vertical) can be integrated along the beamline to calculate the power on the collimator.

As an example, consider the Bypass line CXBP21 with a 4 GeV beam of 1.2 MW, a vacuum pressure of 1×10^{-7} Torr of CO (2 atoms/molecule, Z ~ 7), an average R_{12}^2 = ½*300*400 m2 over 1000 meters, and ax = 4mm → 1.1W of beam hitting the collimator. The expression is relatively large due to the large beta-functions in the Bypass line and would be 10x smaller in a line with beta-functions of ~30 meters assuming the collimator gaps are also scaled. Finally, as discussed below in Section 3, the expression above should be modified from $Z^2 \rightarrow Z(Z+1)$ to include the effect of the atomic electrons.

2. Bremsstrahlung – Nucleus and Atomic Electrons:

Next, we'll calculate the effect of inelastic scattering with the beam gas that leads to bremsstrahlung. We'll only consider relatively soft collisions with emissions of a few % of the beam energy using the complete screening model and only keeping the leading terms [2,3]. In this regime, the differential cross section for scattering off the nucleus and atomic electrons is roughly:

$$\frac{d\sigma}{d\delta} \approx \frac{16\alpha r_0^2}{3}\frac{(1-\delta)}{\delta} Z(Z+1.35) ln\left(\frac{183}{Z^{1/3}}\right)$$

where the factor of 1.35 includes the effect of the atomic electrons, δ is the fractional energy loss, and α is the fine structure constant. To calculate the loss on the energy collimators which have an energy aperture of $\pm\Delta E/E$, we integrate along the beamline:

$$\frac{dN}{cdt} = N_b n_{gas} \int_{\frac{\Delta E}{E}}^{1} \frac{d\sigma}{d\delta} d\delta = N_b n_{gas}(s) \frac{16\alpha r_0^2}{3} Z(Z+1.35) ln\left(\frac{183}{Z^{\frac{1}{3}}}\right)\left[\frac{\Delta E}{E} - ln\frac{\Delta E}{E} - 1\right]$$

This expression can be integrated along the beamline to find the total scattering. Assuming a 3% energy aperture after 2000 meters of Bypass line with the same pressure as above, the power on the collimator would be a few mW and the bremsstrahlung sources can be ignored.

3. Atomic electrons – Elastic scattering:

Scattering events with atomic electrons result in both angular and energy transfer [4]. Assuming small momentum transfers, the differential cross section is:

$$\frac{d\sigma}{d\delta} \approx \frac{2\pi r_0^2}{\gamma}\frac{Z}{\delta^2}$$

where δ is the relative energy loss and the scattering angle is $\theta = \mathrm{sqrt}(2\delta/\gamma)$ assuming $\theta,\delta \ll 1$. The scattering term is equivalent to changing Z^2 to $Z(Z+1)$ in the Coulomb nuclear scattering rate described in Section 1 above which is 10 ~ 15% increase in the scattering rates.

The energy loss due to the atomic electrons increases the power on the energy collimators by a few 10's of μW and is negligible.

4. Thermal Photons [5]:

In the warm sections of the beamline, there will be a large number of thermal photons with which the beam can scatter. The density and distribution can be estimated from Planck's Law for black-body radiation:

$$\frac{dn_\gamma}{\hbar d\omega} = \frac{(\hbar\omega)^2}{\pi^2 c^3 \hbar^6 (e^{\omega/kT} - 1)}$$

and the total number of photons per cubic meter is: $2\times10^7\ T^3 m^{-3}$ where T is in Kelvin. At 300°K, the photon density is comparable to the residual gas atomic density at a pressure of 10^{-8} Torr and the average energy of the photons is $\omega_{ave} = 2.7kT \sim 70$ meV. In the lab frame, the maximum scattering energy change is equal to the energy of a backscattered photon which is:

$$E_\gamma \leq E_0 4\gamma^2$$

where E_0 is the photon energy and E_γ is the backscattered energy which is less than 0.5% of the 4 GeV beam. Similarly, the maximum angular deflection is given by the transverse momentum transfer in the beam rest frame and is:

$$x' = \frac{p_x}{p_0} \leq \frac{2E_0}{mc^2} \ ,$$

corresponding to a 0.3 μrad deflection. The scattering rate is given by the Thomson cross-section however, even in the worst case, these scattering events will not deflect the beam out of the energy or transverse aperture and can be ignored.

5. Vacuum Pressure:

Typical Ultra-High Vacuum (UHV) is dominated by H_2 and then has significant CO and traces of other gasses while vacuum gauges measure N_2-equivalent pressure. The conversion between partial pressures and N_2-equivalent is described in Ref. [6]. The scattering calculations should be based on the sum of the partial pressures for the different gas components. The 'Effective Pressure' defined in Ref. [7] provides an approximate evaluation of the weighted impact of the different gas species that is based largely on bremsstrahlung. Another model would be to base the effective pressure on the Coulomb scattering, the more important source of scattering in the LCLS-II, however the simplest solution is to perform the correct calculations based on partial pressures. Examples of partial pressures and the gas composition can be found in Ref. [6].

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[1] Jackson, "Classical Electrodynamics," Chap 13.7, Wiley, New York (1975).
[2] Bethe and Ashkin, "Passage of Radiation through Matter," Section 2A, *Experimental Nuclear Physics, Vol 1.*, edited E. Serge, Wiley, New York (1953).
[3] Porter, NIM, A302 (1991) 209.

[4] Bethe and Ashkin, "Passage of Radiation through Matter," Section 2C, *Experimental Nuclear Physics, Vol 1*., edited E. Serge, Wiley, New York (1953).
[5] Telnov, NIM, A260 (1987) 304.
[6] D. Gill, "Residual Gas PRD Effective Pressure to Nitrogen Equivalent Pressure Conversion," LCLSII-2.1-EN-0329 (2015).
[7] J. Welch, "Residual Gas Physics Requirements," LCLSII-2.1-PR-0234 (2014).