Analytical Solutions for Heat Distribution in KB Mirror Systems

SLAC NATIONAL ACCELERATOR LABORATORY

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Introduction

The high power X-ray laser produced by LCLS II will be focused using a pair of focusing optics (KB mirrors), which must be cooled to prevent deformation and damage. There are several feasible cooling configurations; performance varies with choice. The goal of this project is to analytically solve for the heat distribution across a mirror for each cooling configuration. The analytical solutions will allow the most effective cooling system to be chosen.

Keywords: LCLS II, KB mirrors, cooling, heat distribution

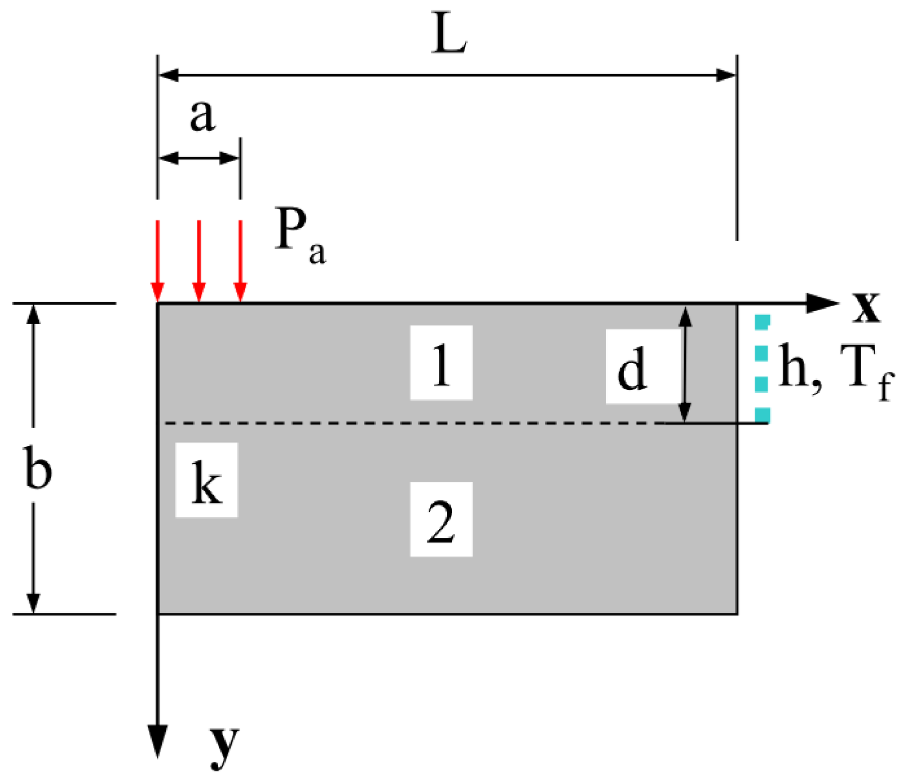


Figure 1*: 2D Steady State Top-Up-Side Cooling. This configuration shows one quarter of the full surface of the mirror being analyzed. The beam width is **a**, and the mirror is cooled across **d**, and **k** is the mirror's thermal conductivity.

Background*

The starting point in solving these heat distributions analytically is the General Fourier Heat Conduction Equation, assuming constant thermal conductivity (k):

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}\right)+q=\rho k\frac{\partial T}{\partial t}$$

For steady state problems, this can be simplified to:

$$-k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=q$$

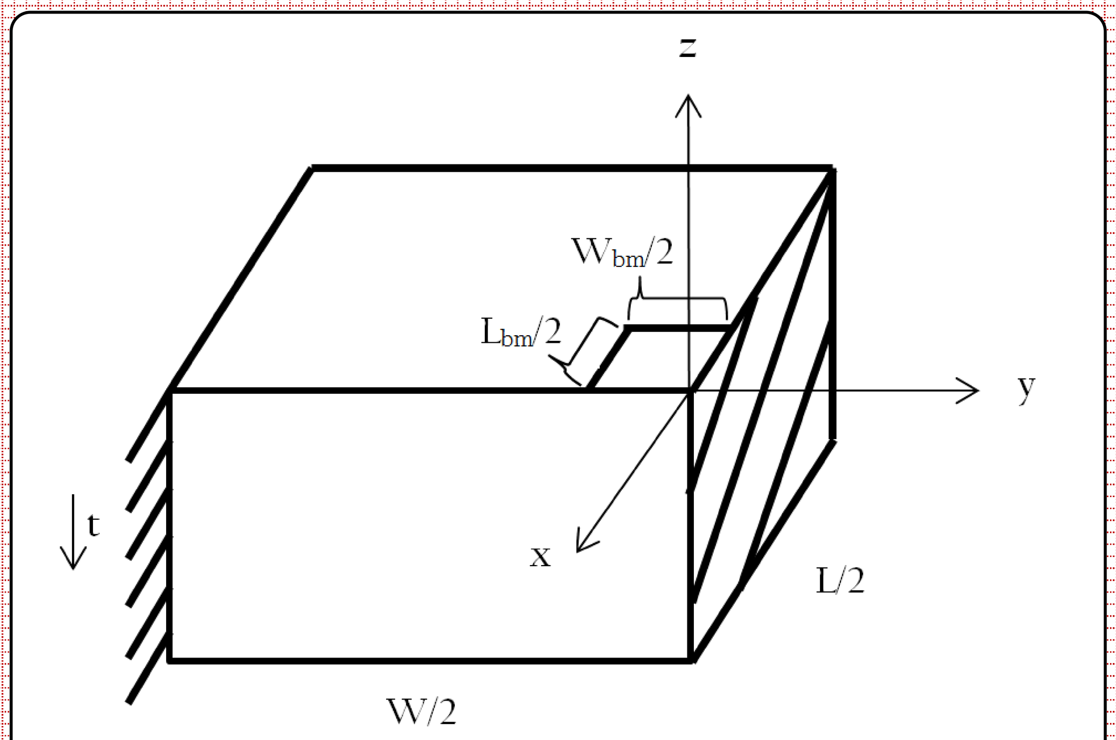


Figure 2: 3D Steady State Full Side Cooling. This image shows one quarter of the full volume of the mirror being analyzed. The beam here has a length, L_{bm}, and a width, W_{bm}. To simplify this problem to the 2D version, simply set L_{bm} equal to the length of the mirror.

Results

2D Steady State Solution for Top-Up-Side Cooling:

$$T_{1\,(y=d)}=T_{2\,(y=d)}$$

$$\sum_{n=1}^{\infty}[C_n\sinh(\beta_n d)+D_n\cosh(\beta_n d)]\cdot\cos(\beta_n x)=G_o+\sum_{n=1}^{\infty}G_n\cdot\frac{\cosh(\omega_n(b-d))}{\sinh(\omega_n b)}\cdot\cos(\omega_n x)$$

$$\left.\frac{\partial T_1}{\partial y}\right|_{(y=d)}=\left.\frac{\partial T_2}{\partial y}\right|_{(y=d)}$$

$$\sum_{n=1}^{\infty}[C_n\cosh(\beta_n d)+D_n\sinh(\beta_n d)]\beta_n\cdot\cos(\beta_n x)=\sum_{n=1}^{\infty}G_n\omega_n\cdot\frac{\sinh(\omega_n(b-d))}{\sinh(\omega_n b)}\cdot\cos(\omega_n x)$$

$$C_n=-\frac{4P_a}{k\beta_n}\cdot\frac{\sin(\beta_n a)}{\sin(2\beta_n L)+2L\beta_n}$$

3D Steady State Solution for Full Side Cooling:

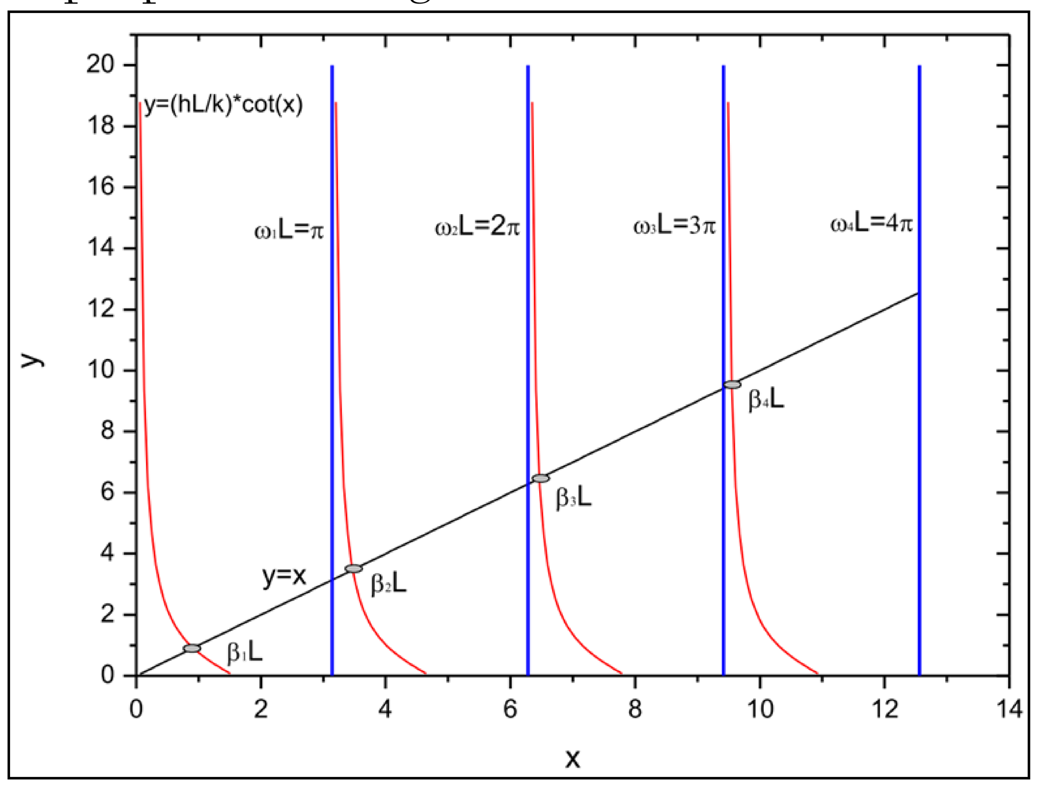
$$T_{steady}(x,y,z)=T_f+\sum_{n=1}^{\infty}\frac{4P_o}{kLW\alpha_n^2}\cdot\sin\left(\frac{\alpha_n L_{bm}}{2}\right)\cdot\left[W_{bm}-\frac{8\sin(\beta_m y)\sin\left(\frac{\beta_m W_{bm}}{2}\right)}{\beta_m(\beta_m W+\sin(\beta_m W))}\right]\cdot\frac{\cosh(\alpha_n(t-z))}{\sinh(\alpha_n t)}\cdot\cos(\alpha_n x)$$

$$+\sum_{m=1}^{\infty}\frac{4P_o L_{bm}}{kL\beta_m}\cdot\frac{\sin(\beta_m y)}{(\beta_m W+\sin(\beta_m W))}\cdot\frac{\cosh(\beta_m(t-z))}{\sinh(\beta_m t)}\cdot\cos(\beta_m y)$$

$$+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{16P_o}{kL\alpha_n\gamma_{nm}}\cdot\frac{\sin\left(\frac{\alpha_n L_{bm}}{2}\right)\sin\left(\frac{\beta_m W_{bm}}{2}\right)}{(\beta_m W+\sin(\beta_m W))}\cdot\frac{\cosh(\gamma_{nm}(t-z))}{\sinh(\gamma_{nm} t)}\cdot\cos(\alpha_n x)\cdot\cos(\beta_m y)$$

Results Cont.

Top-Up-Side Cooling Solution Details:



$$\beta_n=\frac{h}{k}\cot(\beta_n L),\, n=1,2,3,\ldots$$

$$\omega_n=\frac{n\pi}{L},\, n=0,1,2,3,\ldots$$

Figure 3*: Plot of the two eigenvalues present in the top-up-side cooling solution.

- It can be seen that as $n\to\infty$, $\beta_n\to\omega_n$
- Depending upon the requested precision of the answer, $\beta_n\approx\omega_n$ after n > n'
- Let n' be defined as the n such that $1-\frac{\omega_{n'}}{\beta_{n'}}\leq a$, where a is the requested precision
- Numerically solve for all n<n', then use the approximation $\beta_n\approx\omega_n$ to analytically solve for all n≥n'
- Given L, h, k, and the requested precision, one can use MATLAB to determine the value of n'

Example:

- L = 50mm, h = .005mm, and k = .138 W/m*K
- Precision = .1
- Calculated: n' = 3
- Solution: Numerically solve for G_1 through G_3 and D_1 through D_3, then use the approximation $\beta_n\approx\omega_n$ to analytically solve for G_n and D_n, n=3,4,5,...

Conclusions

After solving the 2D steady state versions of bottom cooling and full side cooling, I was able to derive an analytical solution for 2D steady state top-up-side cooling. Then after solving the 3D steady state solution to bottom cooling, I successfully derived a 3D steady state solution for full side cooling. The next steps in this project are to find the 3D transient state solutions for bottom and full side cooling. Then, to find the 3D transient state solution for top-up-side cooling. Finally, it is essential to use MATLAB to model all the solutions that were obtained, and check them against numerical solutions to verify their legitimacy.

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*Lin Zhang, SLAC Engineering Seminar, 2015

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