# Analytical Solutions for Temperature Distribution in Blocks and Application for LCLS II Optics

SLAC NATIONAL ACCELERATOR LABORATORY

Jiya Janowitz[1,2], Lin Zhang[2*]

[1]Physics Department, Material Science and Engineering Department, Stanford University, 450 Serra Mall, Stanford, CA 94305, USA

[2]Linac Coherent Light Source, SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, CA 94025, USA

[*]zhanglin@slac.stanford.edu

## Introduction

The high power X-ray laser produced by LCLS II will be focused using optics that must be cooled to prevent deformation and damage. There are several feasible cooling configurations; performance varies with choice. The goal of this project is to analytically solve for the heat distribution across a block for each cooling configuration. The analytical solutions will allow the most effective cooling system to be chosen for LCLS II optics.

Keywords: LCLS II, heat distribution, cooling, optics

## Background*

The starting point in solving these heat distributions analytically is the General Fourier Heat Conduction Equation, assuming constant thermal conductivity (k):

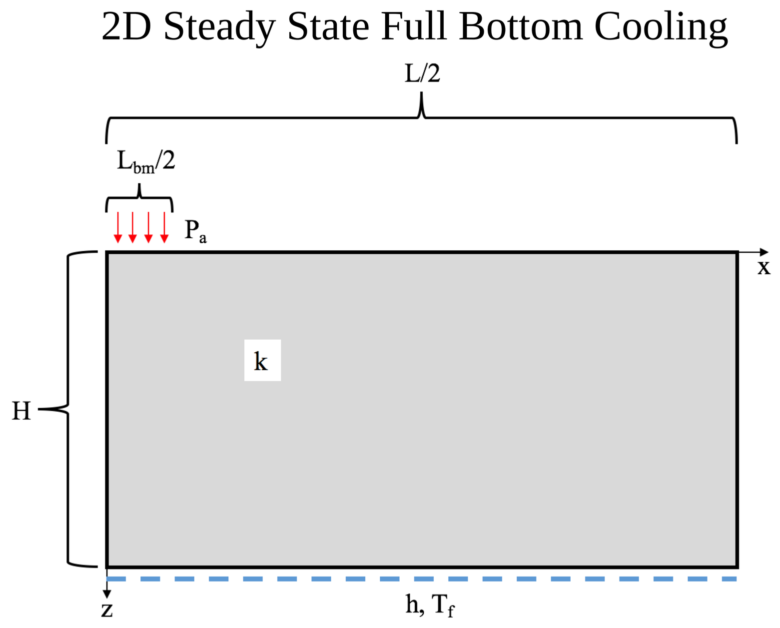
$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}\right)+q=\rho k\frac{\partial T}{\partial t}$$

For steady state problems, this can be simplified to:

$$-k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=q$$

## Problem Setup

### 2D Steady State Full Bottom Cooling



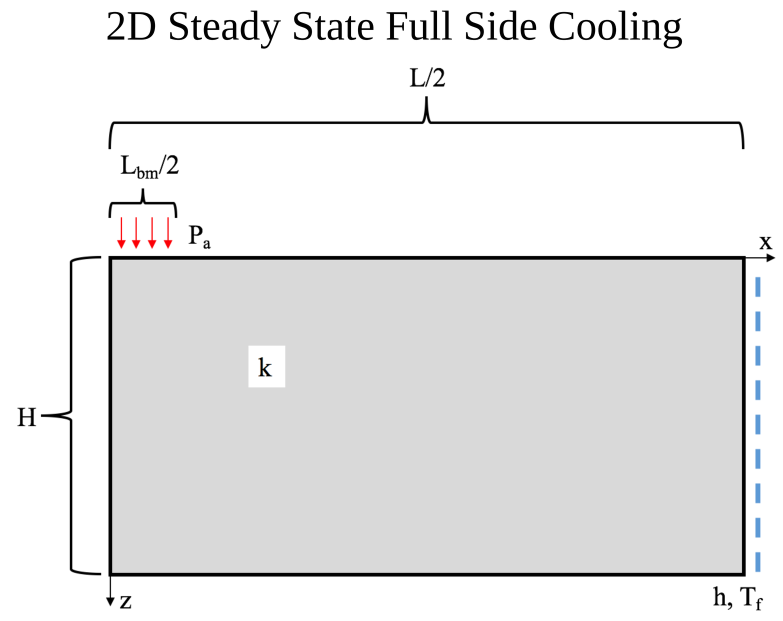

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}=0$$

$$\left.\frac{\partial T}{\partial x}\right|_{(x=0,\frac{L}{2})}=0$$

$$\left.\frac{\partial T}{\partial z}\right|_{(z=0)}=-\frac{P_a}{k}f(x)$$

$$\left.\frac{\partial T}{\partial z}\right|_{(z=H)}=-\frac{h}{k}(T-T_f)$$

### 2D Steady State Full Side Cooling




$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}=0$$

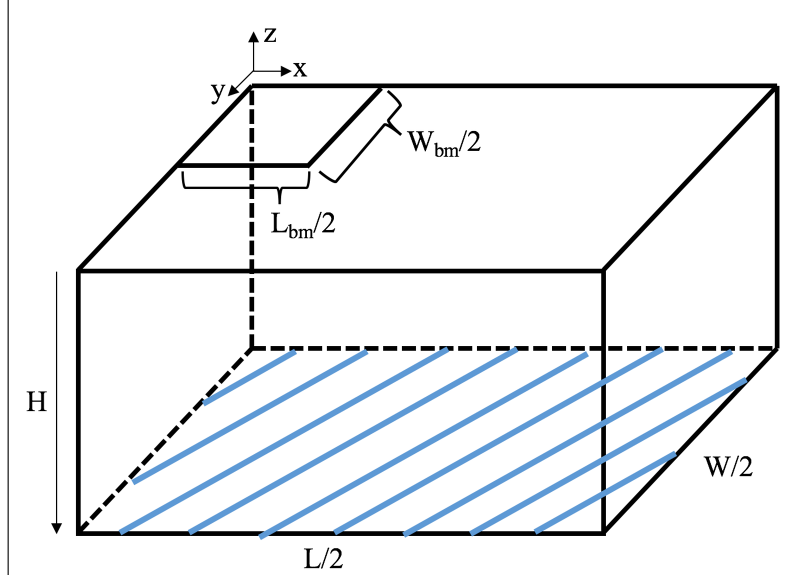
$$\left.\frac{\partial T}{\partial x}\right|_{(x=0)}=0$$

$$\left.\frac{\partial T}{\partial x}\right|_{(x=\frac{L}{2})}=-\frac{h}{k}(T-T_f)$$

$$\left.\frac{\partial T}{\partial z}\right|_{(z=0)}=-\frac{P_a}{k}f(x)$$

$$\left.\frac{\partial T}{\partial z}\right|_{(z=H)}=0$$

### 3D Steady State Bottom Cooling




$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0$$

$$\left.\frac{\partial T}{\partial x}\right|_{(x=0,\frac{L}{2})}=0$$

$$\left.\frac{\partial T}{\partial y}\right|_{(y=0,\frac{W}{2})}=0$$

$$\left.\frac{\partial T}{\partial z}\right|_{(z=0)}=-\frac{P_a}{k}f(x)$$

$$\left.\frac{\partial T}{\partial z}\right|_{(z=H)}=-\frac{h}{k}(T-T_f)$$

## Results

### 2D SS BC

$$T(x,z)=T_f+\frac{L_{bm}P_a}{Lk}\left(\frac{k}{h}+H-z\right)+\sum_{n=1}^{\infty}\frac{4P_a}{Lk\alpha_n^2}\cdot sin\left(\frac{\alpha_n L_{bm}}{2}\right)\frac{cosh\big(\alpha_n(H-z)\big)+\frac{h}{\alpha_n k}sinh\big(\alpha_n(H-z)\big)}{sinh(\alpha_n H)+\frac{h}{\alpha_n k}cosh(\alpha_n H)}\cdot cos(\alpha_n x)$$

### 2D SS SC

$$T(x,z)=T_f+\sum_{n=1}^{\infty}\frac{4P_a}{k\alpha_n}\cdot\frac{sin\left(\frac{\alpha_n L_{bm}}{2}\right)}{(sin(\alpha_n L)+L\alpha_n)}\cdot\frac{cosh\big(\alpha_n(H-z)\big)}{sinh(\alpha_n H)}\cdot cos(\alpha_n x)$$

### 3D SS BC

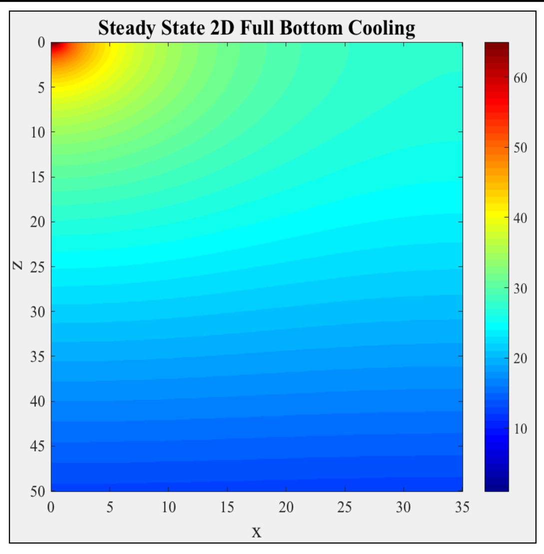
$$T(x,y,z)=T_f+\frac{P_aW_{bm}L_{bm}}{kWL}\cdot\left[\frac{k}{h}+H-z\right]+\sum_{n=1}^{\infty}\frac{4P_aW_{bm}}{kWL\alpha_n^2}\cdot sin\left(\frac{\alpha_n L_{bm}}{2}\right)\cdot\frac{cosh\big(\alpha_n(H-z)\big)+\frac{h}{\alpha_n k}sinh\big(\alpha_n(H-z)\big)}{sinh(\alpha_n H)+\frac{h}{\alpha_n k}cosh(\alpha_n H)}\cdot cos(\alpha_n x)$$

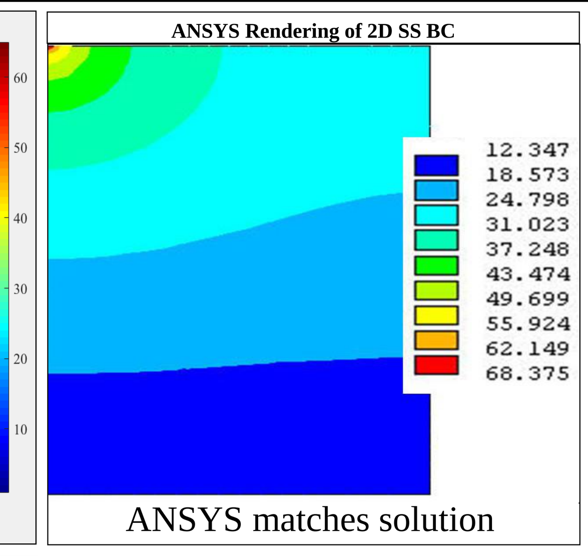
$$+\sum_{m=1}^{\infty}\frac{4P_aL_{bm}}{kWL\beta_m^2}\cdot sin\left(\frac{\beta_m W_{bm}}{2}\right)\cdot\frac{cosh\big(\beta_m(H-z)\big)+\frac{h}{\beta_m k}sinh\big(\beta_m(H-z)\big)}{sinh(\beta_m H)+\frac{h}{\beta_m k}cosh(\beta_m H)}\cdot cos(\beta_m y)$$

$$+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{16P_a}{kWL\alpha_n\beta_m\mu_{nm}}\cdot sin\left(\frac{\alpha_n L_{bm}}{2}\right)\cdot sin\left(\frac{\beta_m W_{bm}}{2}\right)\cdot\frac{cosh\big(\mu_{nm}(H-z)\big)+\frac{h}{\mu_{nm}k}sinh\big(\mu_{nm}(H-z)\big)}{sinh(\mu_{nm}H)+\frac{h}{\mu_{nm}k}cosh(\mu_{nm}H)}\cdot cos(\alpha_n x)\cdot cos(\beta_m y)$$

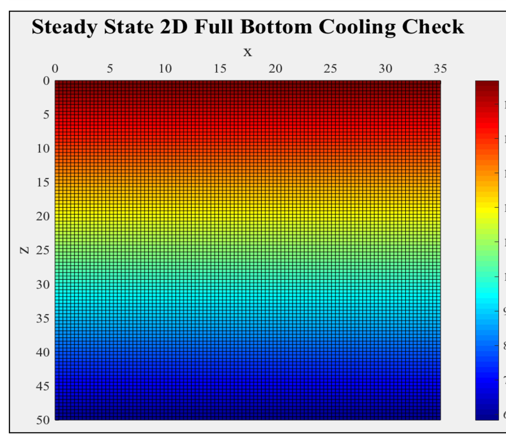
Background Image: 2D Steady State Bottom Cooling

## MATLAB Results




ANSYS matches solution

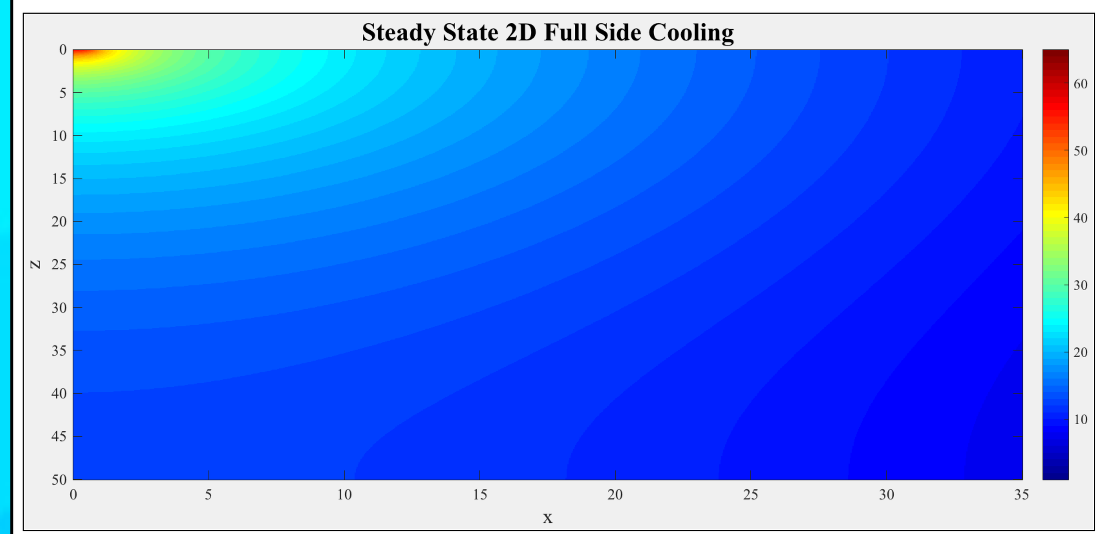


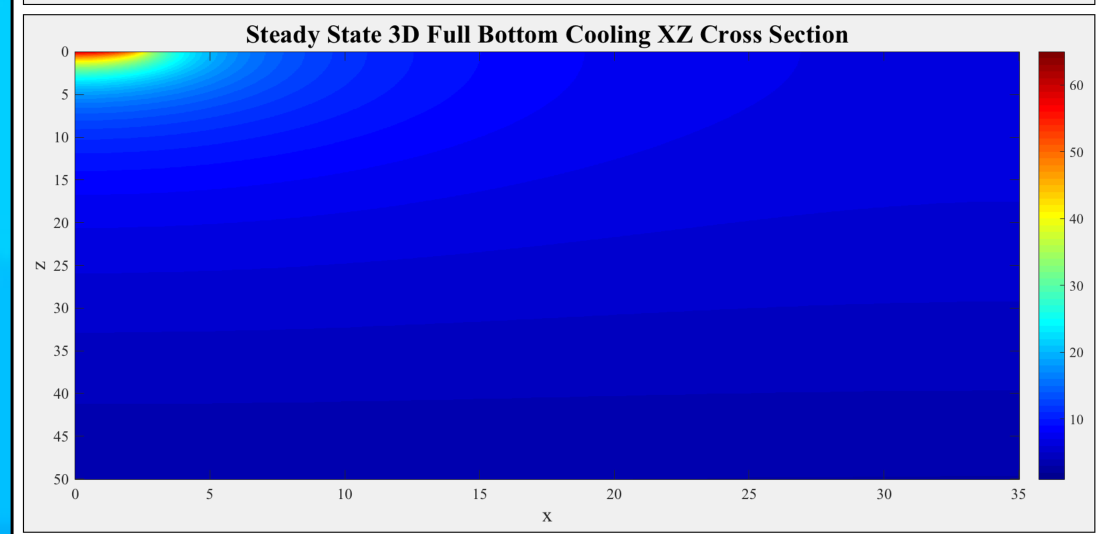

$$k\frac{T_1-T_2}{H}=P_c$$

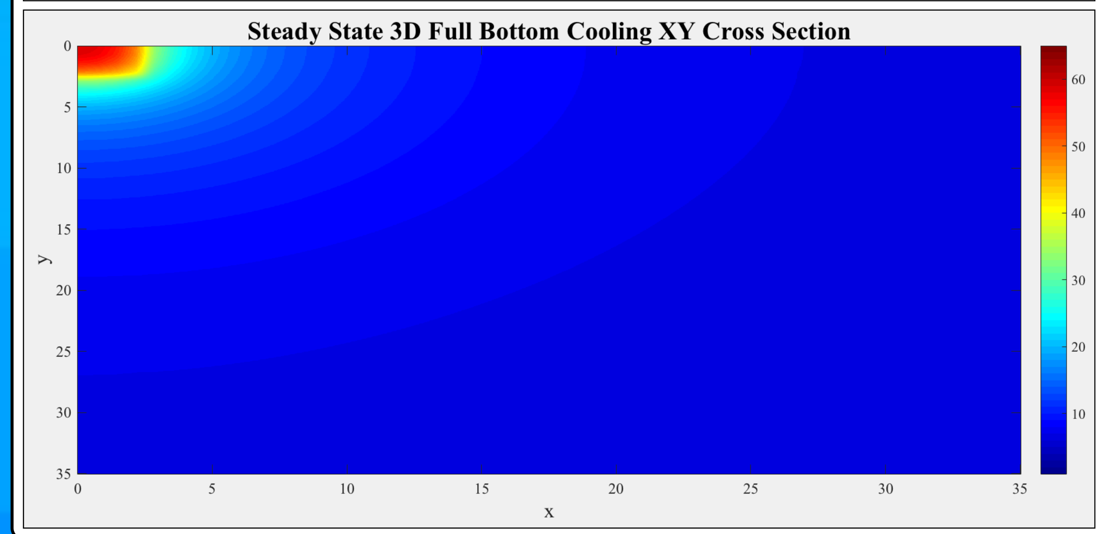
$$(0.148)\frac{1569-583.4}{50}=2.9174$$

$$P_c=2.917=P_r\ \checkmark$$

After setting $L_{bm}$ = L and making the problem 1D, the calculated value for P was equal to the real value of P = 2.917.




## Conclusions

Analytical solutions were obtained for 2D and 3D, steady state and transient, bottom and side cooling. To verify the legitimacy of these solutions, MATLAB was used to graph the results. Bottom cooling solutions were confirmed by extending the size of the beam to the surface of the mirror, making the problem 1D. The next steps in this project are to continue coding and confirming analytical solutions for more complex problems.

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*Lin Zhang, SLAC Engineering Seminar, 2015

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