Interstage Optics Design

for a PWFA Linear Collider

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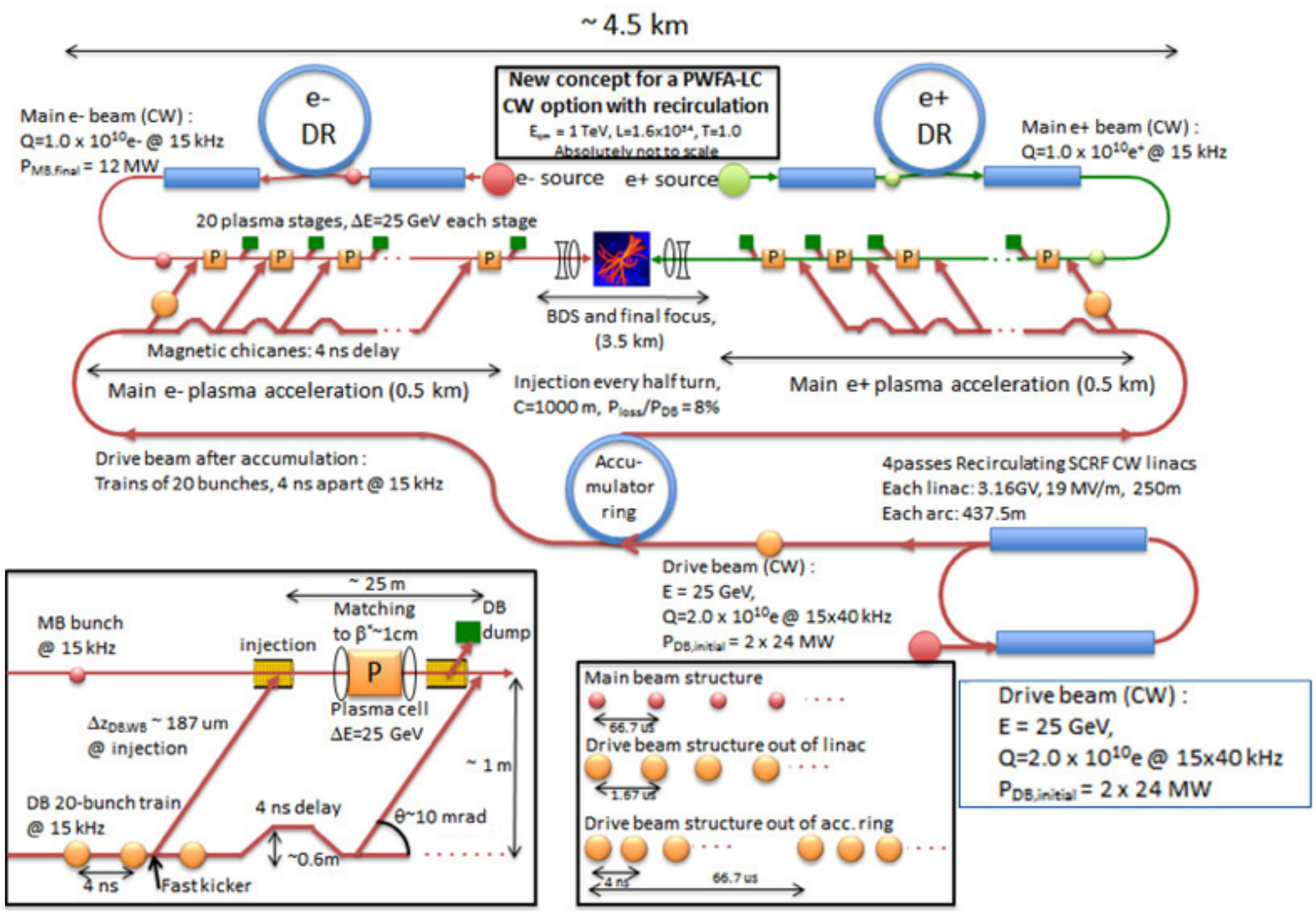
Proposed layout of a PWFA Linear Collider*

- Basis for this study + source of parameters.



* E Adli, JP Delahaye, et al. (presented at IPAC'13)

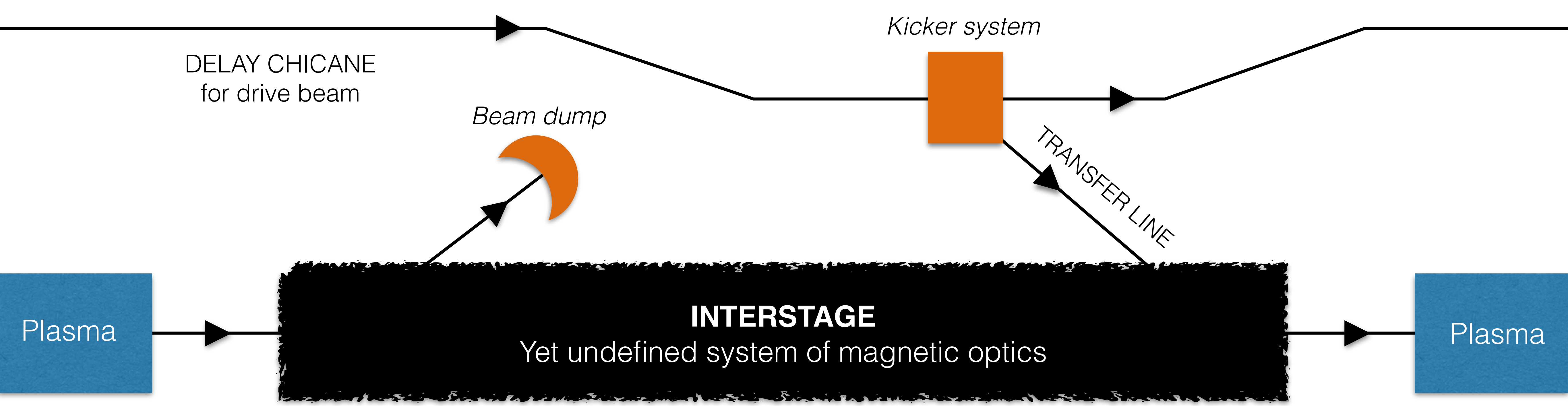
Interstage optics

- Problem at hand:

Switch old with fresh drive beam,

keep the main beam focused

and preserve its emittance.



Formal requirements

- Drive beam injection/extraction

- Collinearity

- Beta function matching

$$\beta_x(L) = \beta_y(L) = \beta_{mat}$$
$$\alpha_x(L) = \alpha_y(L) = 0$$

- Dispersion cancellation

$$D_x(L) = D'_x(L) = 0$$

- Limit bunch lengthening (R_{56})

$$R_{56}(L) \ll \frac{\sigma_z}{\delta} \approx 1 \text{ mm}$$

- Emittance preservation
  – limit chromaticity
  – limit synchrotron radiation

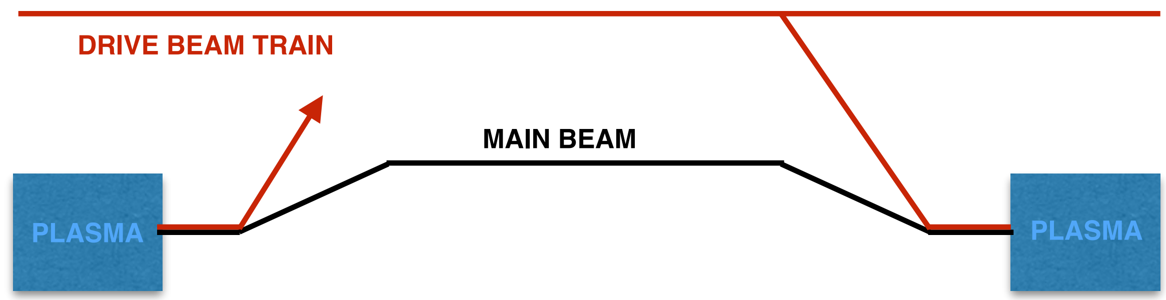
$$\frac{\Delta\epsilon}{\epsilon}(L) \ll 1\%$$

- Minimize length subject to all the above

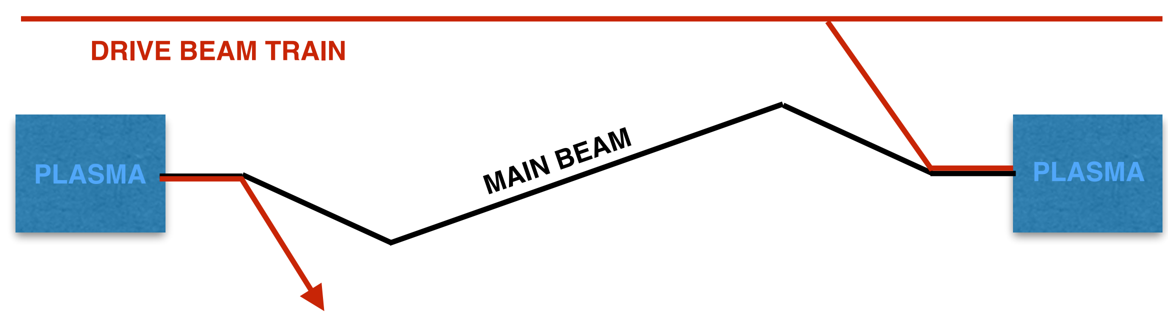
Injection/extraction of drive beams

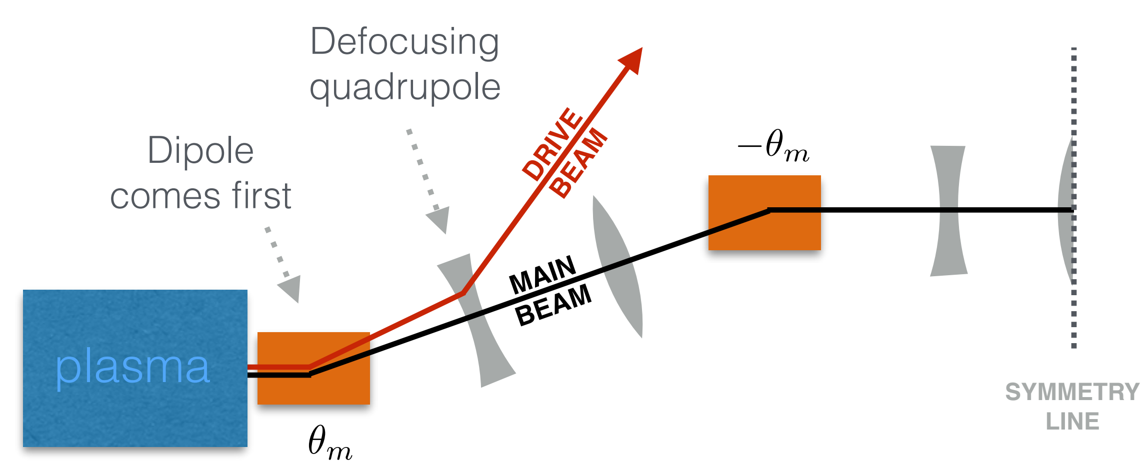
- Using dipoles to create dispersion: **Separate beams spatially by energy.**
- Injection and extraction are **symmetric processes ⇒ mirror symmetric lattice.**
- Injection/extraction **dipoles must be first and last magnets,** as main beam quads would destroy the drive beam.
- Important: **Dipoles do not scale with main beam energy** (only drive beam energy).
- Defines regimes:
  – **Low energy** ($E_{main} \approx E_{drive}$) ⇒ Dipoles "visible"
  – **High energy** ($E_{main} >> E_{drive}$) ⇒ Dipoles "invisible"
- Scalings:
  – **Dipole field & length:** L_{dipole}, $B \sim$ constant
  – **SR power:** $P_{SR} \sim E_{main}^2$
  – **Main beam dispersion:** $D_x \sim L_{interstage}/E_{main} \sim 1/\sqrt{E_{main}}$
  (assuming focusing ⇒ $L_{interstage} \sim \sqrt{E_{main}}$)

Option 1: **"C-chicane"**
Weaker bending



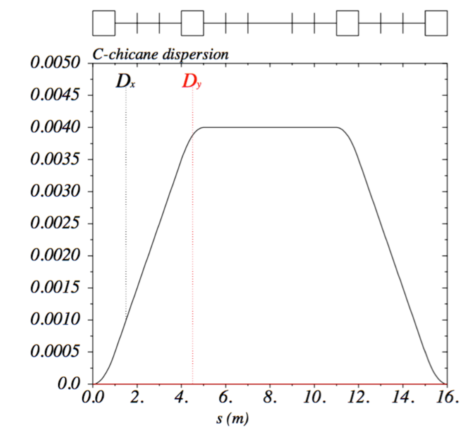
Option 2: **"S-chicane"**
Stronger bending, more space for beam dump.



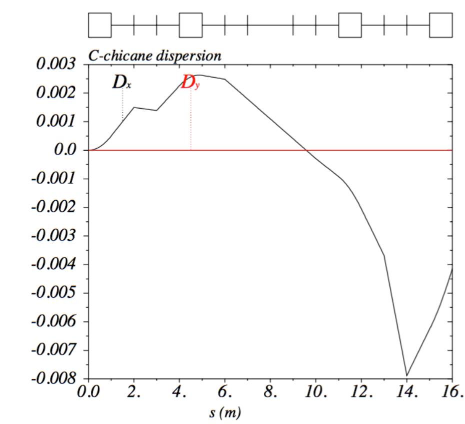


Dispersion cancellation

- Both S- and C-chicane has inherent dispersion cancellation. Quadrupoles change this.
- Cancel by:
  - **Matching quadrupoles** (not independent of beam matching)
  - **Inserting extra dipoles** (independent of beam matching)
- Low energy regime (large main beam dispersion) may require second order dispersion cancellation.
- Becomes less important with higher energies: **Scaling:** $\mathbf{D_x(s) \sim 1/\sqrt{E_{main}}}$



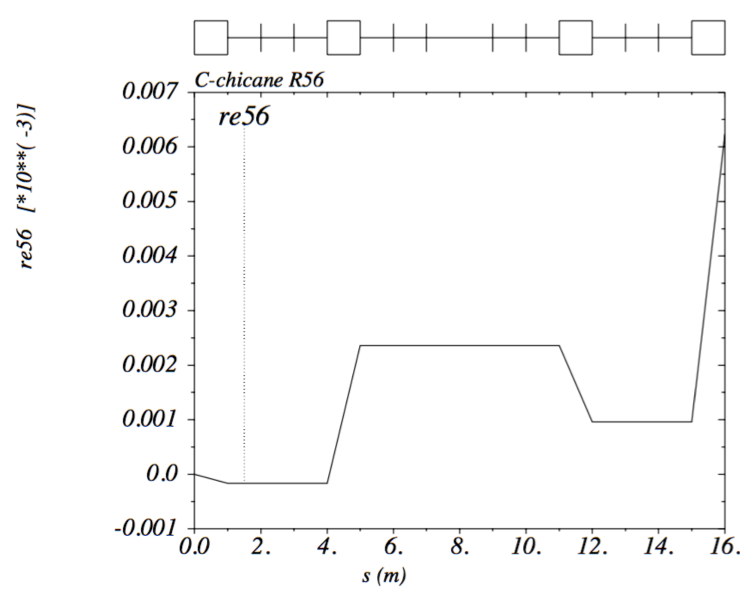
D_x without quads



D_x with quads

Limiting bunch lengthening (R_{56})

- Not big problem due to relatively weak dipoles.
- Becomes easier with higher energy.
  Scaling: $\mathbf{R_{56}(s) \sim D_x(s) \sim 1/\sqrt{E_{main}}}$
- If necessary, a method for matching $R_{56} \approx 0$:
  - **Low dispersion in dipoles.**



R_{56} with quads

$$R_{56}(L) \ll \frac{\sigma_z}{\delta} \approx 1 \text{ mm}$$

Matching beam to plasma

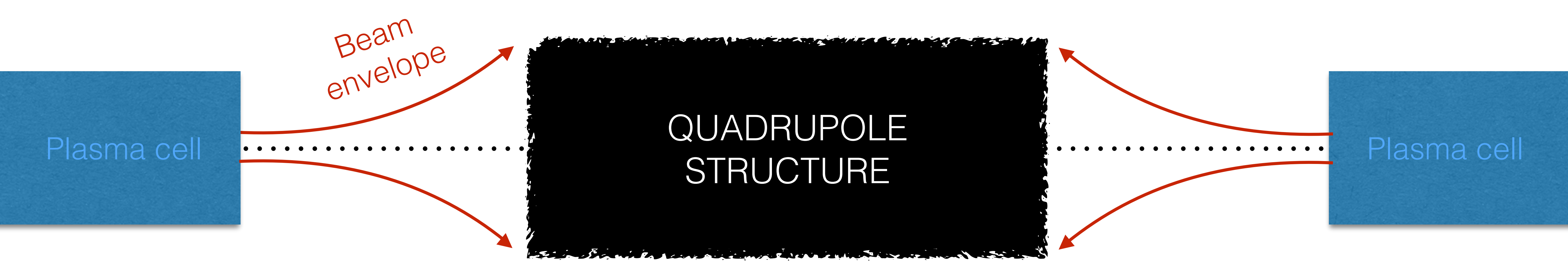
$\alpha_x(L) = \alpha_y(L) = 0 \quad \beta_x(L) = \beta_y(L) = \beta_{mat}$

- Matched beta in a plasma: $\beta_{matched} = \frac{\sqrt{2\gamma}}{k_p} = \sqrt{\frac{2\epsilon_0 E}{n_p e^2}}$ $\Rightarrow$ 3.3 cm @ E = 100 GeV, $n_p = 10^{16}$ cm^{-3}

- Naive matching is simple: **place quadrupoles, match strengths/separations.**

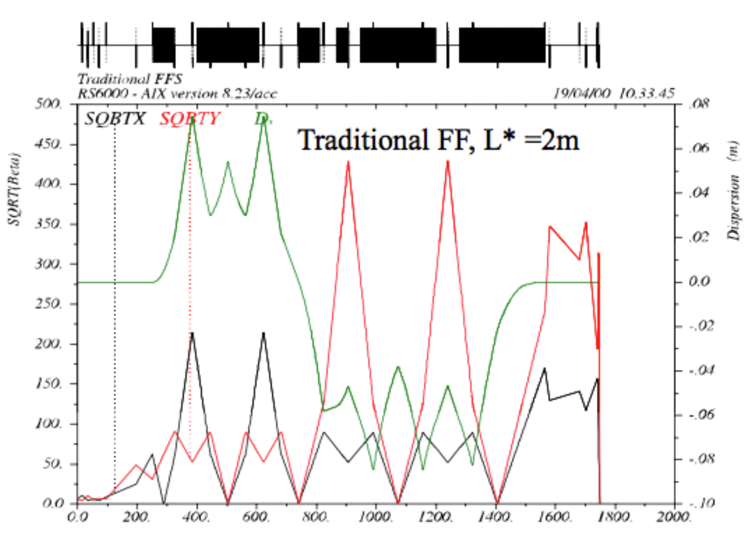
- **Strong plasma channel focusing** $\Rightarrow$ small plasma betas
  $\Rightarrow$ strongly diverging/large beams
  $\Rightarrow$ long/strong quadrupoles
  $\Rightarrow$ **large chromaticity** (big challenge)

- Requirements similar to those of a **final focus system, for every stage.**

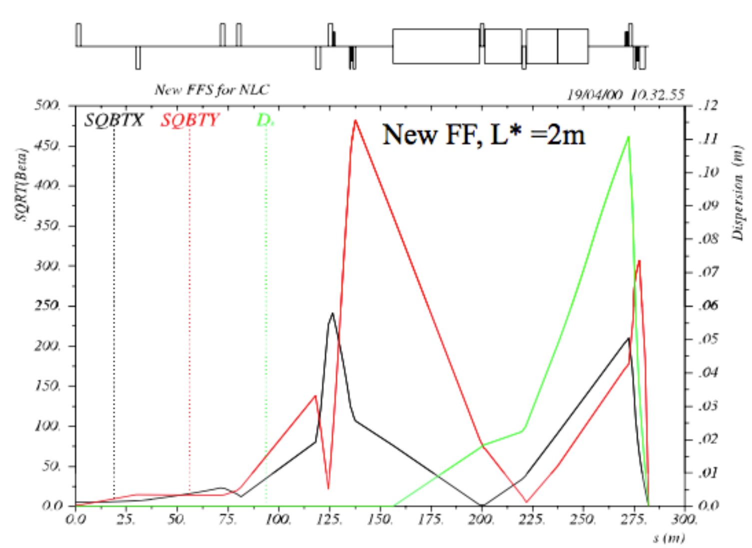


Chromaticity cancellation

- Small beams in plasma lead to large chromaticity.
- PWFALC study assumes a ~1% rms. energy spread.
- Conventionally **corrected using sextupoles.**
  Seen in final focus systems (SLC, ILC…): **very long lattices.**
- We will consider a sextupole solution, and a novel solution.



Traditional FF @ 500GeV ~ 1750 m

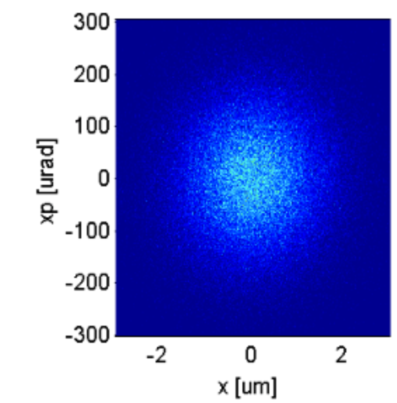


Improved FF @ 500GeV ~ 300 m

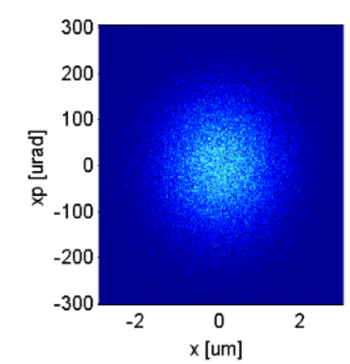
Figures from "Beam Delivery & beam-beam" by **Andrei Seryi (SLAC)*

Example:

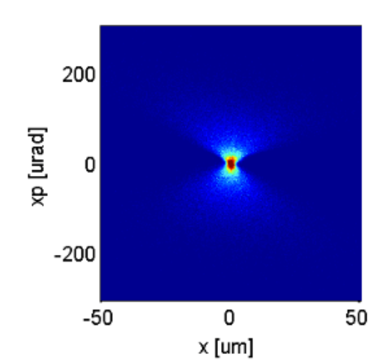
Initial phase space (x, x')



Final phase space (**no energy spread**)



Final phase space (**1% energy spread**)



No chromaticity correction:
Emittance increases by many orders of magnitude

Plasma density ramp

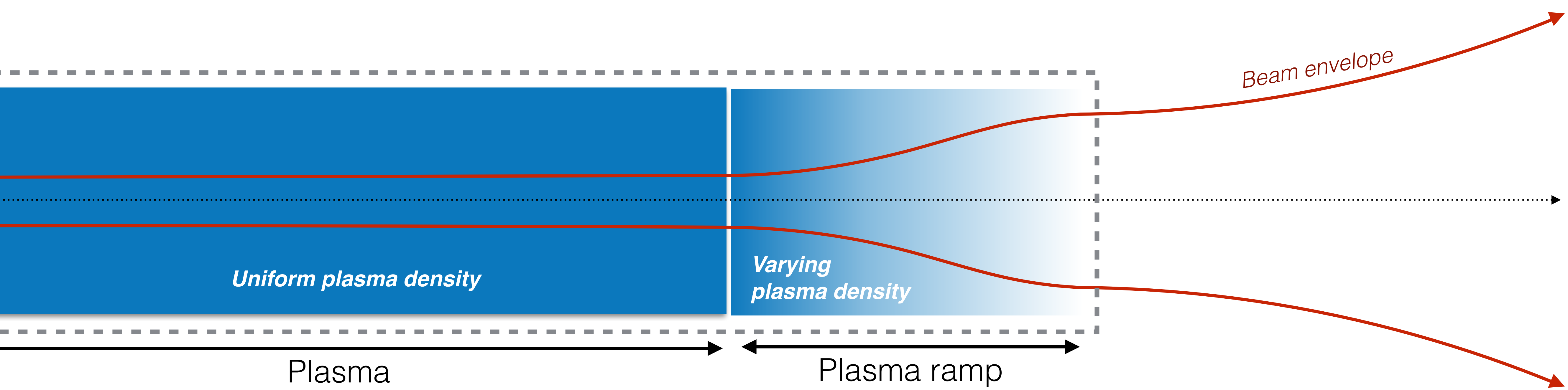
* Plasma ramp paper by Xu et al. (2015): http://arxiv.org/pdf/1411.4386v2.pdf

- A plasma density ramp **mitigates the chromaticity problem.**

- Plasma ramp ⇒ larger/less diverging beam
  ⇒ smaller beam in quadrupoles ⇒ less chromaticity

- Any adiabatic density profile will do. Use pressure gradients or partial laser-ionization.

- **Scaling** (for magnification factor Π_p): $\mathbf{L_{ramp} \sim \Pi_p \sqrt{E_{main}}}$



Conventional solution: Sextupoles

- Sextupole effect is stronger with:
  - **Larger dispersion**
  - **Larger beam size**

- **Long lattices** for geometric term cancellation.

- Introduces **new problems:**

  - Dipoles must ramp with main beam energy
    - ⇒ **Dispersion / R_{56} / SR scales poorly** with energy
  - -I transforms require repeated sections
    - ⇒ "unnecessarily" **long lattices**
  - Thick sextupoles (imperfect -I transforms)
    - ⇒ **geometric errors** (emittance growth)
  - Sextupoles need large beam sizes
    - ⇒ **increased energy spread** from SR (Oide effect)

Sextupole B-fields:

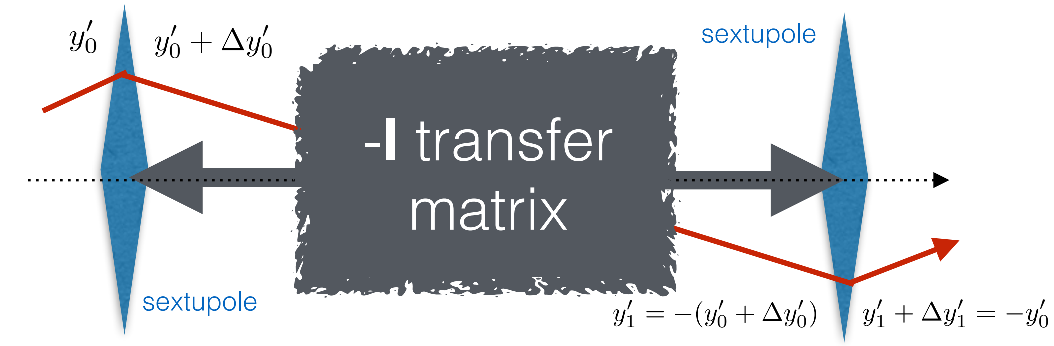
Non-linear geometric terms **NEEDS TO BE SMALL**

Non-linear chromatic term **NEED TO BE CANCELLED**

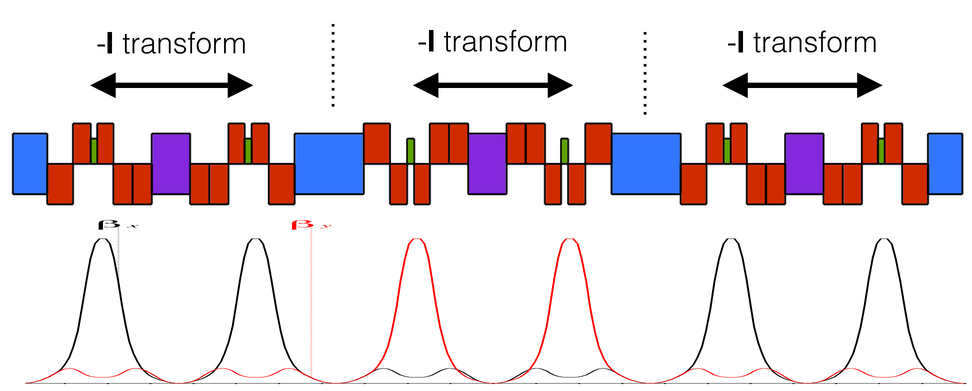
$$B_x \sim xy + \delta D_x y \qquad B_y \sim \frac{1}{2}(x^2 - y^2) + x\delta D_x + \frac{1}{2}\delta^2 D_x^2$$

Linear chromatic terms **CORRECT CHROMATICITY**

Geometric term cancellation:



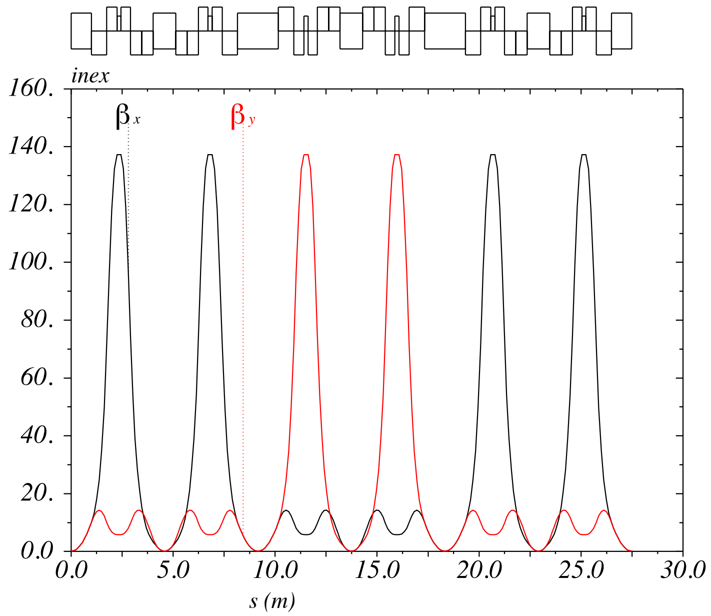
"Working" interstage using sextupoles:



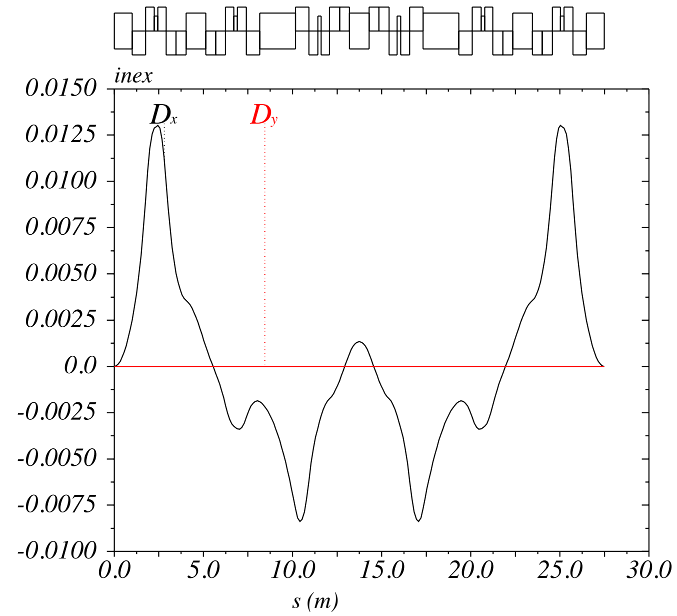
"Working" interstage using sextupoles

100 GeV, β_{mat} = 0.1m

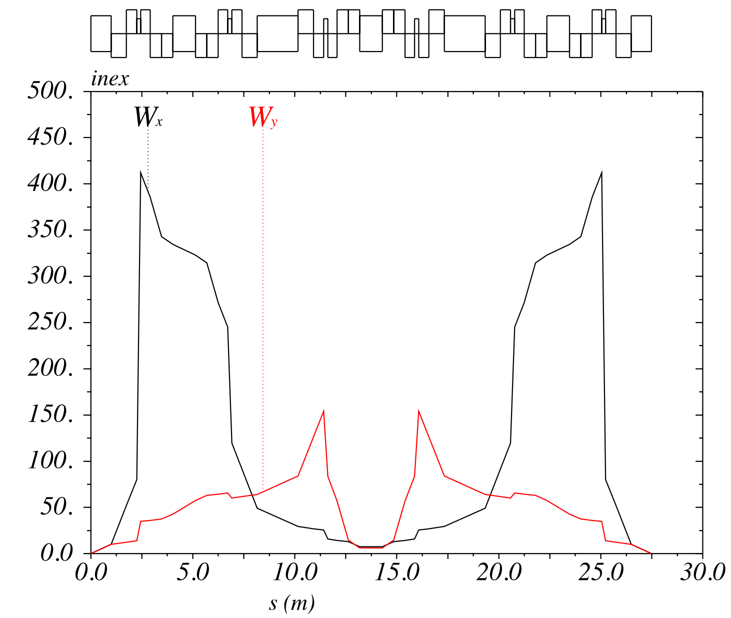
Beta functions



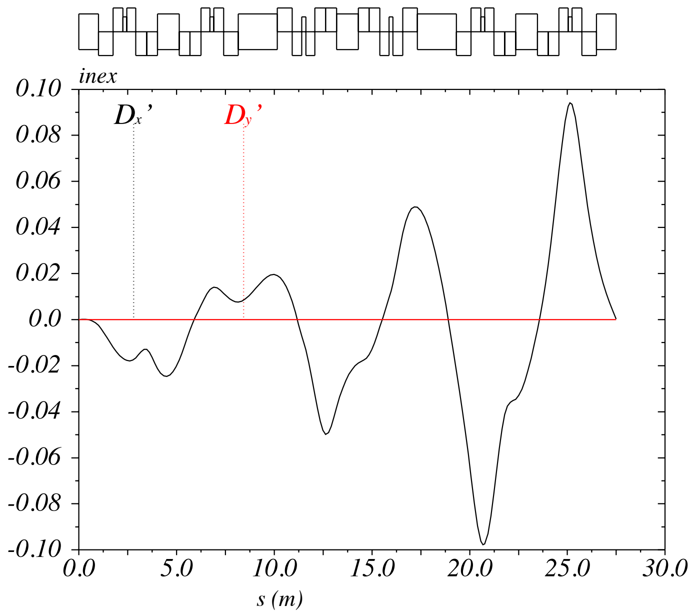
Dispersion



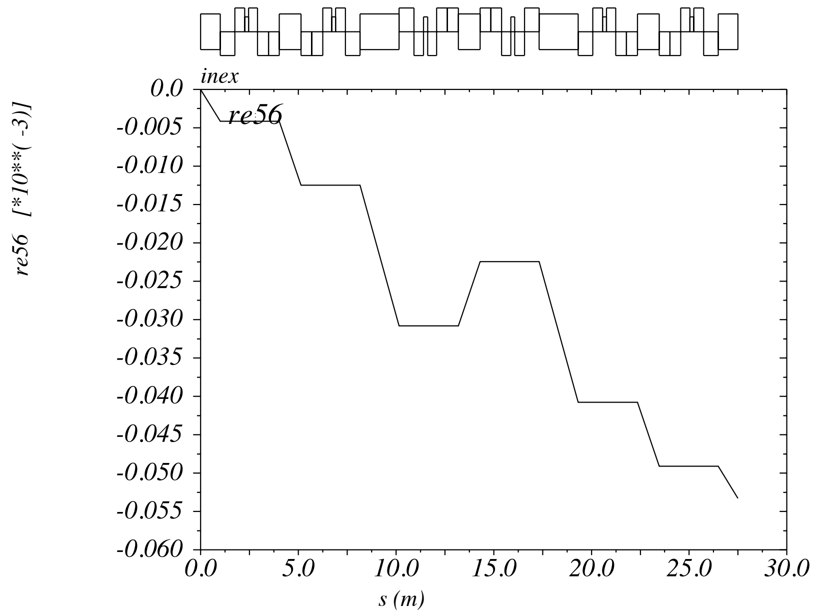
W-functions (chrom.)



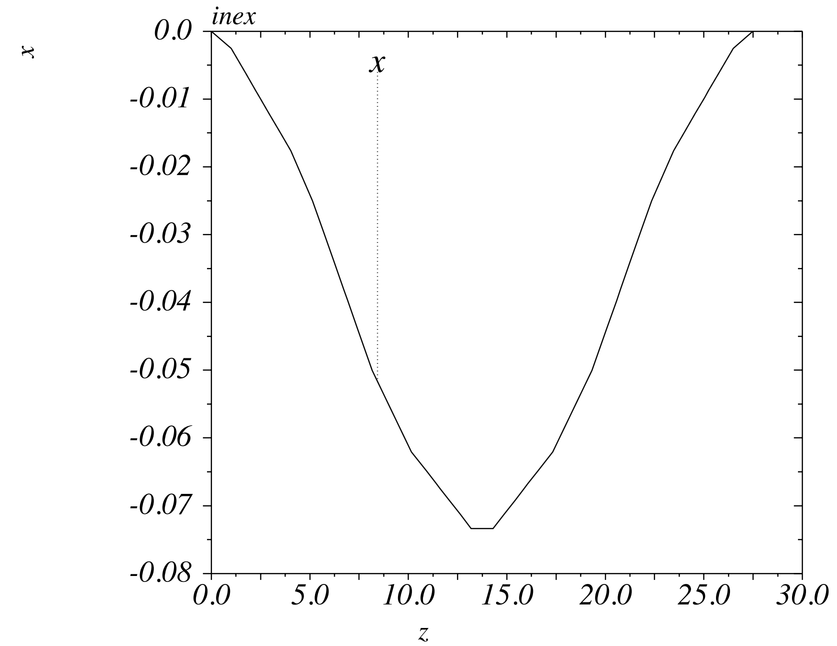
Second order dispersion



R_{56}



Footprint (x-z)



- This solution requires stronger sextupoles than currently manufacturable. Not a conceptual show-stopper.

Novel solution: Getting rid of sextupoles

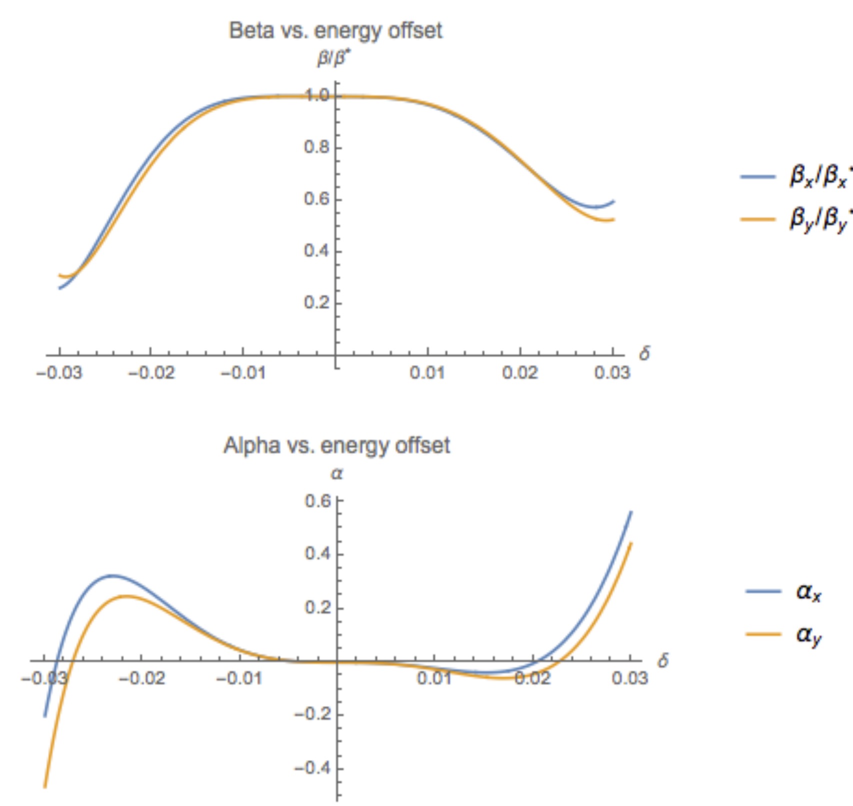
- We are developing a **new method for finding quadrupole-only lattices which cancel chromaticity** to the required order in energy offset.

- Benefits of using quadrupoles only:
  - no geometric terms ⇒ **keeps it linear**
  - no -I transforms ⇒ **much shorter**
  - no ramping dipoles ⇒ **better SR scaling**

- Similar solutions in light optics: **Superachromats** (however, beam optics is x/y-asymmetric).

- Achieved by tailoring the energy offset (δ) expansion of β and α to be flat around δ = 0.

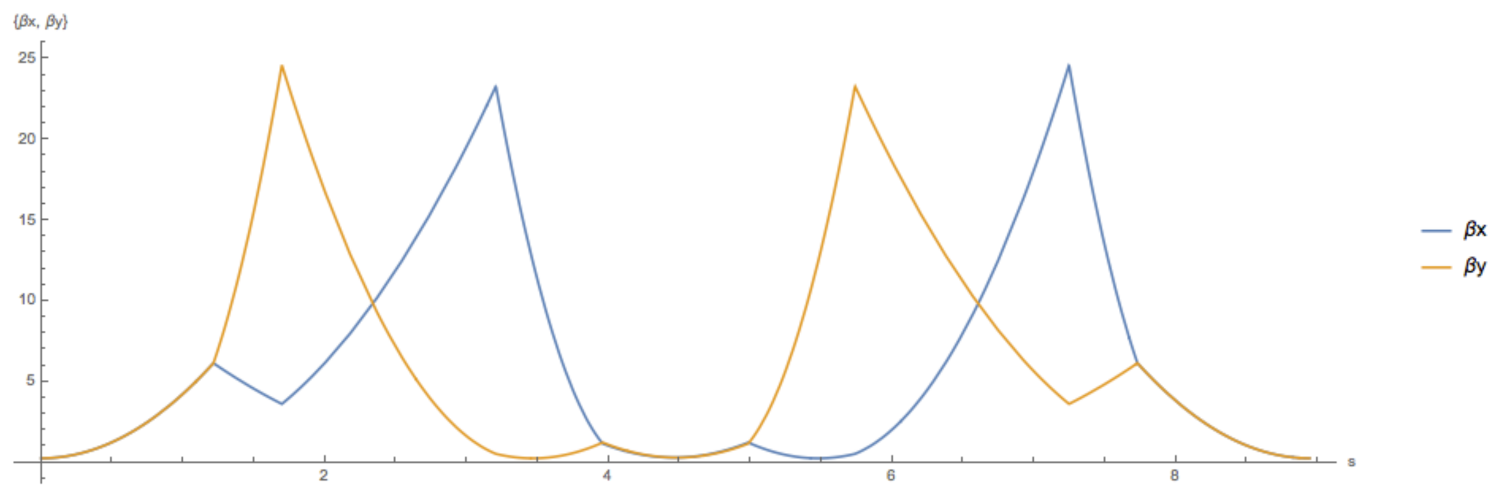
Simpler version used in light optics: "Superachromat" by Carl Zeiss

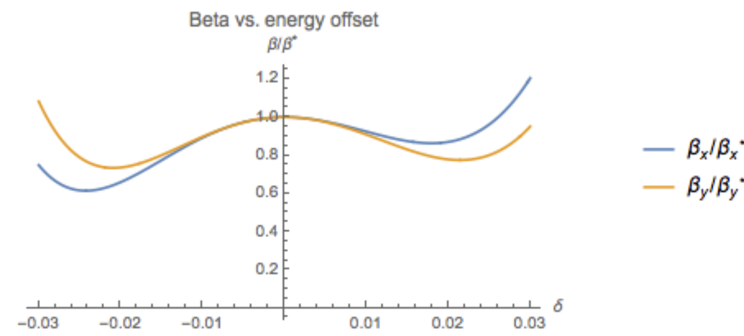


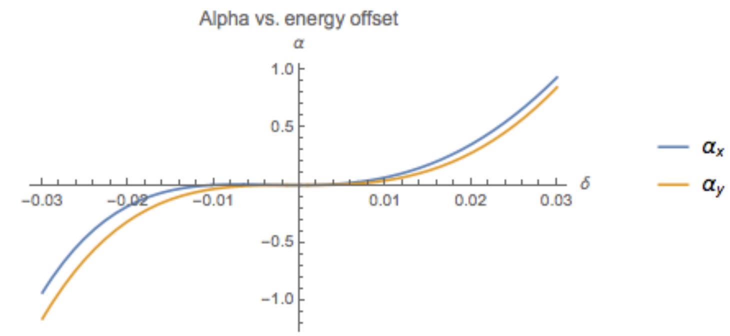
β and α vs. energy offset δ.
Flat regions ⇒ no chromaticity

Examples of chromaticity-free quadrupole lattices

- 8 quads: **cancel chromaticity to 1st order.**



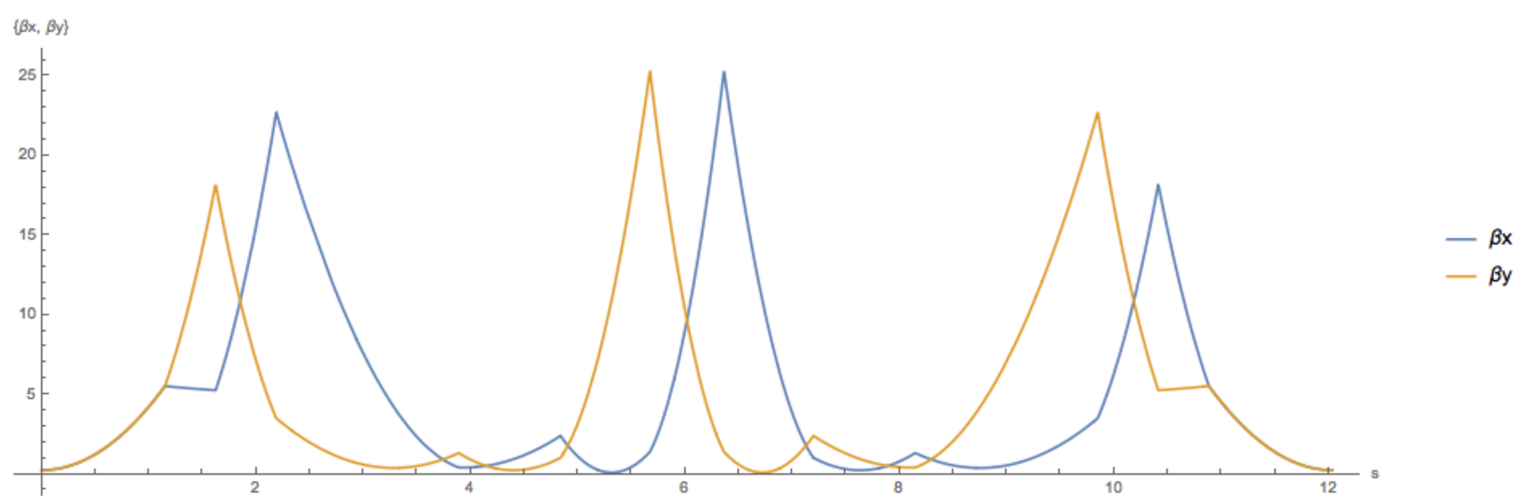


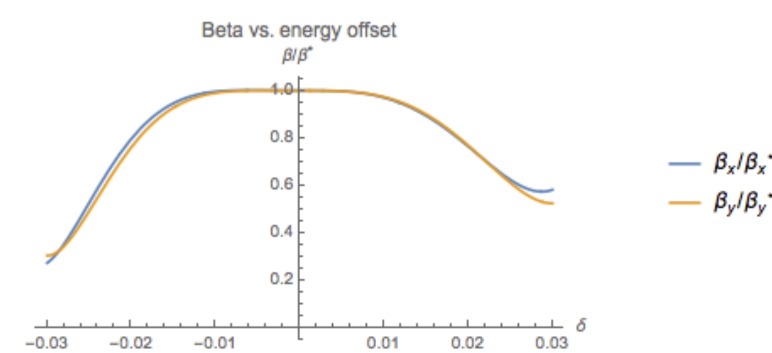


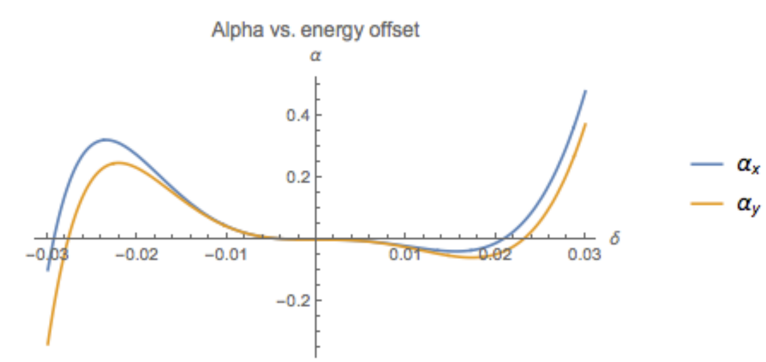
For 0.5% energy spread: $(\Delta\epsilon_x/\epsilon_x,\ \Delta\epsilon_y/\epsilon_y)$ = (0.000781, 0.000782)

For 1.% energy spread: $(\Delta\epsilon_x/\epsilon_x,\ \Delta\epsilon_y/\epsilon_y)$ = (0.0141, 0.0142)

- 12 quads: **cancel chromaticity to 2nd order.**







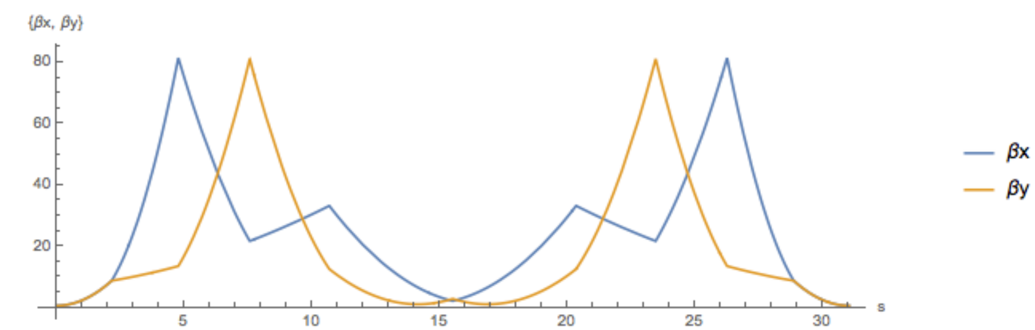
For 0.5% energy spread: $(\Delta\epsilon_x/\epsilon_x,\ \Delta\epsilon_y/\epsilon_y)$ = (0.000186, 0.000186)

For 1.% energy spread: $(\Delta\epsilon_x/\epsilon_x,\ \Delta\epsilon_y/\epsilon_y)$ = (0.0119, 0.0119)

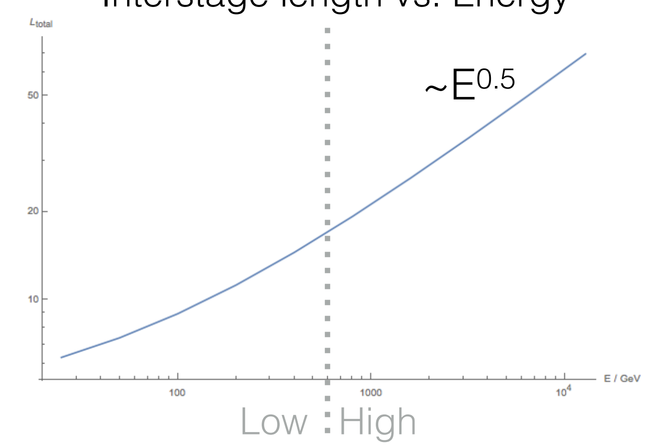
Length estimates and scalings

- **Any beta matching imposes:** $L_{interstage} \sim \sqrt{E_{main}} \Rightarrow L_{collider} \sim E_{main}^{1.5}$
- High energy regime: **retains beta-shape**, constant emittance preservation, good synch. rad. scaling ($P_{SR} \sim E_{main}^2$)
- Low energy regime: complex lattices/many quads (high chrom. correction order), possibly use of sextupoles.
- Interstage **length estimate: ~30 m @ 300 GeV,** (E-spread: **1% rms**, dipole length: **1m**, plasma ramp: **10x**, quads: **150 T/m**, emit. growth: **~1%,** plasma density: **10^{16} cm^{-3}**)
- Note: work in progress.

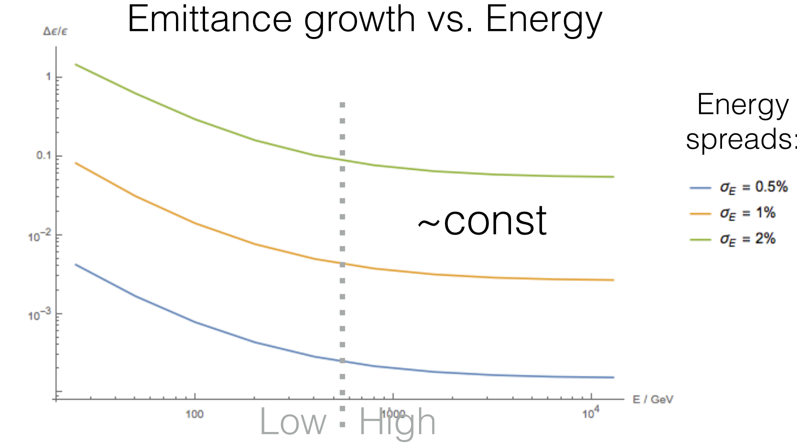
Current best solution (9 quads):



Interstage length vs. Energy



Emittance growth vs. Energy



Approximate scalings for high energy regime:

	Interstage length	**Emittance growth**
Energy	$E_{main}^{0.5}$	const
Quad strength	$g_{max}^{-0.5}$	$g_{max}^{-1.5}$
Energy spread (1st order chromaticity correction)	const	σ_E^4
Ramp magnification	const	Π_p^{-3}

Summary

- **Chromaticity is a big challenge** facing a PWFA interstage.

- Traditional chromaticity correction designs (using **sextupoles**) have **unfavourable energy scaling laws**.

- A new type of lattice:
  Chromaticity-free quadrupole-only lattices have been developed (shorter, less SR, no non-linear terms).

- At high energies, **same optics solution applies to any energy** (length scales, emittance growth is constant)

- PWFA/LWFA **interstage and collider lengths will scale as**:

$$L_{interstage} \sim \sqrt{E_{main}}$$

$$L_{collider} \sim {E_{main}}^{1.5}$$

- Next up:
  - Integration of plasma density ramps
  - Emittance growth studies from SR/misalignment
  - Details of injection/extraction optics