Interstage Optics Design
for a PWFA Linear Collider


Carl A Lindstrøm
PhD Student
University of Oslo, Department of Physics

Advisor: Erik Adli
Proposed layout of a PWFA Linear Collider*

• Basis for this study + source of parameters.

* E Adli, JP Delahaye, et al. (presented at IPAC’13)
Interstage optics

- Problem at hand:

Switch old with fresh drive beam, keep the main beam focused and preserve its emittance.
Formal requirements

- Drive beam injection/extraction
- Collinearity
- Beta function matching
- Dispersion cancellation
- Limit bunch lengthening ($R_{56}$)
- Emittance preservation
  - limit chromaticity
  - limit synchrotron radiation
- Minimize length subject to all the above

\[
\begin{align*}
\beta_x(L) &= \beta_y(L) = \beta_{\text{mat}} \\
\alpha_x(L) &= \alpha_y(L) = 0 \\
D_x(L) &= D'_x(L) = 0 \\
R_{56}(L) &\ll \frac{\sigma_z}{\delta} \approx 1 \text{ mm} \\
\frac{\Delta \epsilon}{\epsilon}(L) &\ll 1%
\end{align*}
\]
Injection/extraction of drive beams

- Using dipoles to create dispersion: Separate beams spatially by energy.

- Injection and extraction are symmetric processes ⇒ mirror symmetric lattice.

- Injection/extraction dipoles must be first and last magnets, as main beam quads would destroy the drive beam.

- Important: Dipoles do not scale with main beam energy (only drive beam energy).

- Defines regimes:
  - Low energy ($E_{\text{main}} \approx E_{\text{drive}}$) ⇒ Dipoles “visible”
  - High energy ($E_{\text{main}} \gg E_{\text{drive}}$) ⇒ Dipoles “invisible”

- Scalings:
  - Dipole field & length: $L_{\text{dipole}}$, $B \sim \text{constant}$
  - SR power: $P_{\text{SR}} \sim E_{\text{main}}$
  - Main beam dispersion: $D_x \sim L_{\text{interstage}}/E_{\text{main}} \sim 1/\sqrt{E_{\text{main}}}$
    (assuming focusing ⇒ $L_{\text{interstage}} \sim \sqrt{E_{\text{main}}}$)

Option 1: “C-chicane”
Weaker bending

Option 2: “S-chicane”
Stronger bending, more space for beam dump.

[Diagrams of chicane options]
Dispersion cancellation

- Both S- and C-chicane has inherent dispersion cancellation. Quadrupoles change this.

- Cancel by:
  - Matching quadrupoles  (not independent of beam matching)
  - Inserting extra dipoles  (independent of beam matching)

- Low energy regime (large main beam dispersion) may require second order dispersion cancellation.

- Becomes less important with higher energies:  Scaling:  \( D_x(s) \sim 1/\sqrt{E_{\text{main}}} \)

Limiting bunch lengthening (\( R_{56} \))

- Not big problem due to relatively weak dipoles.

- Becomes easier with higher energy.  Scaling:  \( R_{56}(s) \sim D_x(s) \sim 1/\sqrt{E_{\text{main}}} \)

- If necessary, a method for matching \( R_{56} \approx 0 \):
  - Low dispersion in dipoles.
Matching beam to plasma

- Matched beta in a plasma: \( \beta_{\text{matched}} = \frac{\sqrt{2\gamma}}{k_p} = \sqrt{\frac{2e_0E}{n_p e^2}} \Rightarrow 3.3 \text{ cm } @ E = 100 \text{ GeV}, n_p = 10^{16} \text{ cm}^{-3} \)

- Naive matching is simple: place quadrupoles, match strengths/separations.

- Strong plasma channel focusing \( \Rightarrow \) small plasma betas
  \( \Rightarrow \) strongly diverging/large beams
  \( \Rightarrow \) long/strong quadrupoles
  \( \Rightarrow \) large chromaticity (big challenge)

- Requirements similar to those of a final focus system, for every stage.
Chromaticity cancellation

- Small beams in plasma lead to large chromaticity.

- PWFALC study assumes a \(~1\%\) rms. energy spread.

- Conventionally corrected using sextupoles. Seen in final focus systems (SLC, ILC…): very long lattices.

- We will consider a sextupole solution, and a novel solution.

*Figures from “Beam Delivery & beam-beam” by Andrei Seryi (SLAC)
Plasma density ramp

- A plasma density ramp mitigates the chromaticity problem.

- Plasma ramp $\Rightarrow$ larger/less diverging beam
  $\Rightarrow$ smaller beam in quadrupoles $\Rightarrow$ less chromaticity

- Any adiabatic density profile will do. Use pressure gradients or partial laser-ionization.

- Scaling (for magnification factor $\Pi_p$): $L_{\text{ramp}} \sim \Pi_p \sqrt{E_{\text{main}}}$

Conventional solution: Sextupoles

- Sextupole effect is stronger with:
  - Larger dispersion
  - Larger beam size

- Long lattices for geometric term cancellation.

- Introduces new problems:
  - Dipoles must ramp with main beam energy
    ⇒ Dispersion / $R_{56}$ / SR scales poorly with energy
  - -I transforms require repeated sections
    ⇒ “unnecessarily” long lattices
  - Thick sextupoles (imperfect -I transforms)
    ⇒ geometric errors (emittance growth)
  - Sextupoles need large beam sizes
    ⇒ increased energy spread from SR (Oide effect)

Sextupole B-fields:

Non-linear geometric terms
NEEDS TO BE SMALL

Non-linear chromatic term
NEED TO BE CANCELLED

$B_x \sim xy + \delta D_{xy}$
$B_y \sim \frac{1}{2} (x^2 - y^2) + x\delta D_x + \frac{1}{2} \delta^2 D_x^2$

Linear chromatic terms
CORRECT CHROMATICITY

Geometric term cancellation:

- Working interstage using sextupoles:
“Working” interstage using sextupoles

- This solution requires stronger sextupoles than currently manufacturable. Not a conceptual show-stopper.
Novel solution: Getting rid of sextupoles

- We are developing a new method for finding quadrupole-only lattices which cancel chromaticity to the required order in energy offset.

- Benefits of using quadrupoles only:
  - no geometric terms ⇒ keeps it linear
  - no -I transforms ⇒ much shorter
  - no ramping dipoles ⇒ better SR scaling

- Similar solutions in light optics: Superachromats (however, beam optics is x/y-asymmetric).

- Achieved by tailoring the energy offset (δ) expansion of β and α to be flat around δ = 0.
Examples of chromaticity-free quadrupole lattices

- 8 quads: cancel chromaticity to 1st order.

\[
\frac{\Delta E_x}{E_x}, \frac{\Delta E_y}{E_y} = (0.000781, 0.000782)
\]
For 1% energy spread: \(\frac{\Delta E_x}{E_x}, \frac{\Delta E_y}{E_y} = (0.0141, 0.0142)\)

- 12 quads: cancel chromaticity to 2nd order.

\[
\frac{\Delta E_x}{E_x}, \frac{\Delta E_y}{E_y} = (0.000186, 0.000186)
\]
For 1% energy spread: \(\frac{\Delta E_x}{E_x}, \frac{\Delta E_y}{E_y} = (0.0119, 0.0119)\)
Length estimates and scalings

• Any beta matching imposes: \( L_{\text{interstage}} \sim \sqrt{E_{\text{main}}} \) \( \Rightarrow \) \( L_{\text{collider}} \sim E_{\text{main}}^{1.5} \)

• High energy regime: retains beta-shape, constant emittance preservation, good synch. rad. scaling (\( P_{\text{SR}} \sim E_{\text{main}}^2 \))

• Low energy regime: complex lattices/many quads (high chrom. correction order), possibly use of sextupoles.

• Interstage length estimate: \( \sim 30 \text{ m} \) @ \( 300 \text{ GeV} \),
  (E-spread: 1% rms, dipole length: \( 1\text{ m} \), plasma ramp: \( 10\times \), quads: \( 150\text{ T/m} \), emit. growth: \( \sim 1\% \), plasma density: \( 10^{16}\text{ cm}^{-3} \))

• Note: work in progress.

 Approximate scalings for high energy regime:

<table>
<thead>
<tr>
<th>Interstage length</th>
<th>Energy</th>
<th>Quad strength</th>
<th>Energy spread (1st order chromaticity correction)</th>
<th>Ramp magnification</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{\text{main}}^{0.5} )</td>
<td>( g_{\text{max}}^{-0.5} )</td>
<td>( \sigma_{E}^{4} )</td>
<td>( \prod_{\text{p}}^{-3} )</td>
<td></td>
</tr>
<tr>
<td>const</td>
<td>( g_{\text{max}}^{-1.5} )</td>
<td>const</td>
<td>const</td>
<td></td>
</tr>
</tbody>
</table>

Current best solution (9 quads):
Summary

- **Chromaticity is a big challenge** facing a PWFA interstage.

- Traditional chromaticity correction designs (using sextupoles) have unfavourable energy scaling laws.

- A new type of lattice: **Chromaticity-free quadrupole-only lattices** have been developed (shorter, less SR, no non-linear terms).

- At high energies, **same optics solution applies to any energy** (length scales, emittance growth is constant)

- PWFA/LWFA interstage and collider lengths will scale as:
  \[ L_{\text{interstage}} \sim \sqrt{E_{\text{main}}} \]
  \[ L_{\text{collider}} \sim E_{\text{main}}^{1.5} \]

- **Next up:**
  - Integration of plasma density ramps
  - Emittance growth studies from SR/misalignment
  - Details of injection/extraction optics