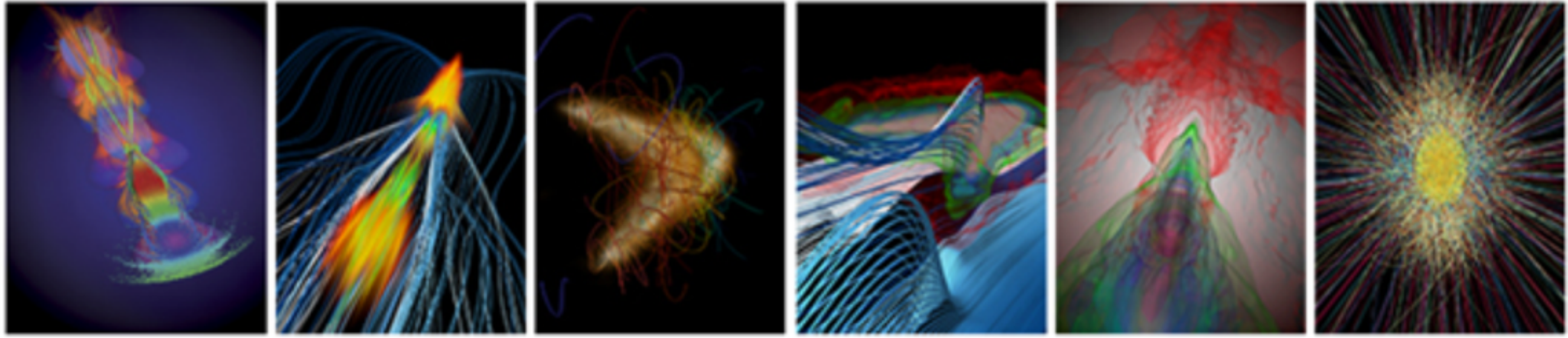

Plasma Ion Motion Induced Emittance Growth In Nonlinear Plasma Wake Field Accelerator

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University of California Los Angeles
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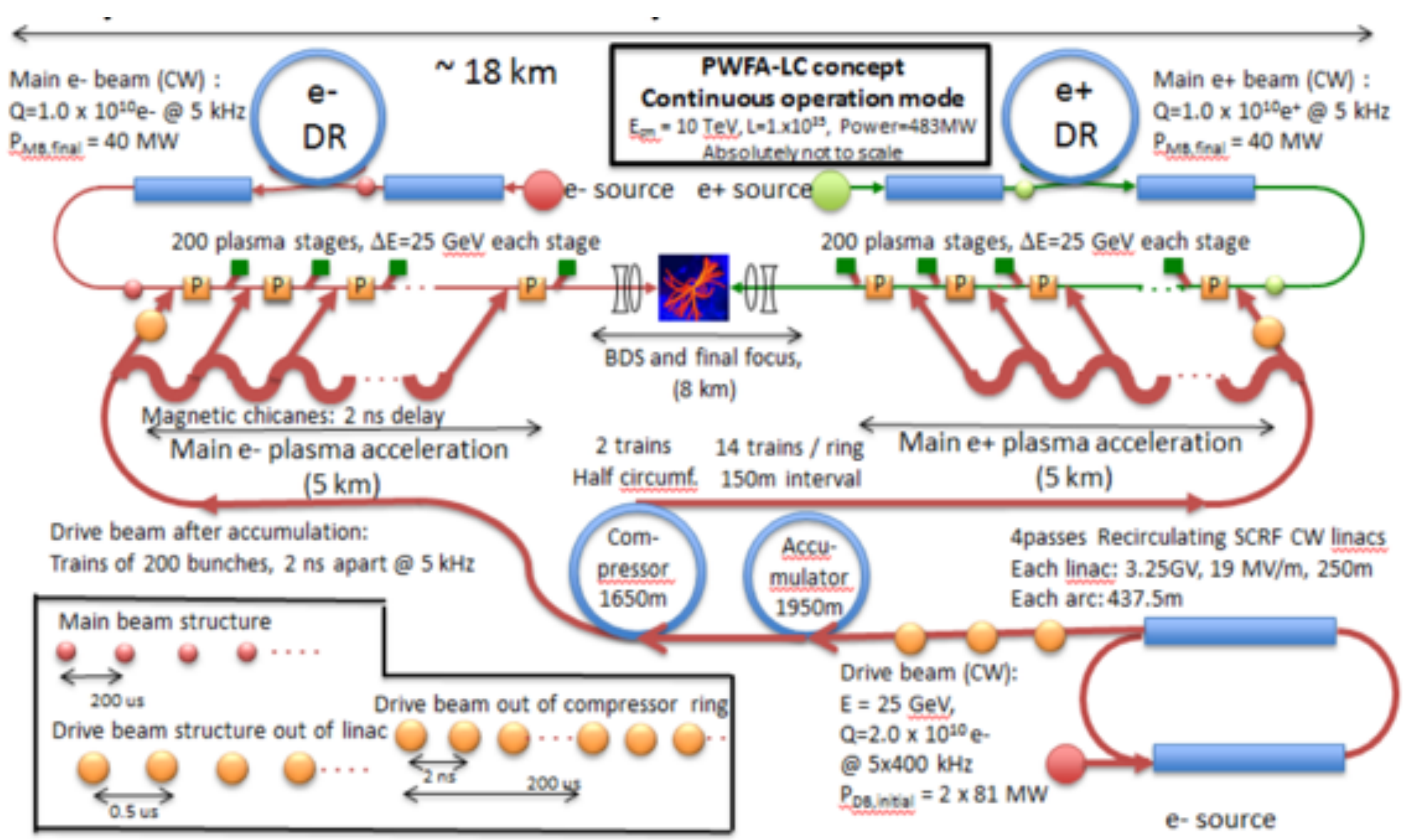
PICKSC
http://picksc.idre.ucla.edu

FACET-II
Facility for Advanced Accelerator Experimental Tests

PWFA Linear Collider

Conceptual design of a 10 TeV PWFA-LC



Luminosity $10^{34}\ \mathrm{cm^{-2}s^{-1}}$

Low ε_N $< 0.1\ \mathrm{mm \cdot mrad}$ $\left(\sqrt{\epsilon_{Nx}\epsilon_{Ny}}\right)$

and

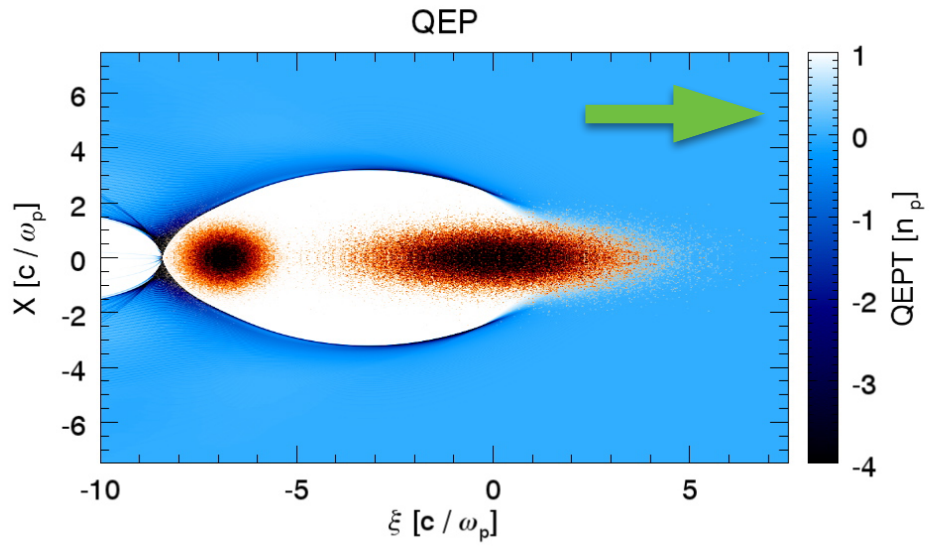
Charge 1 nC

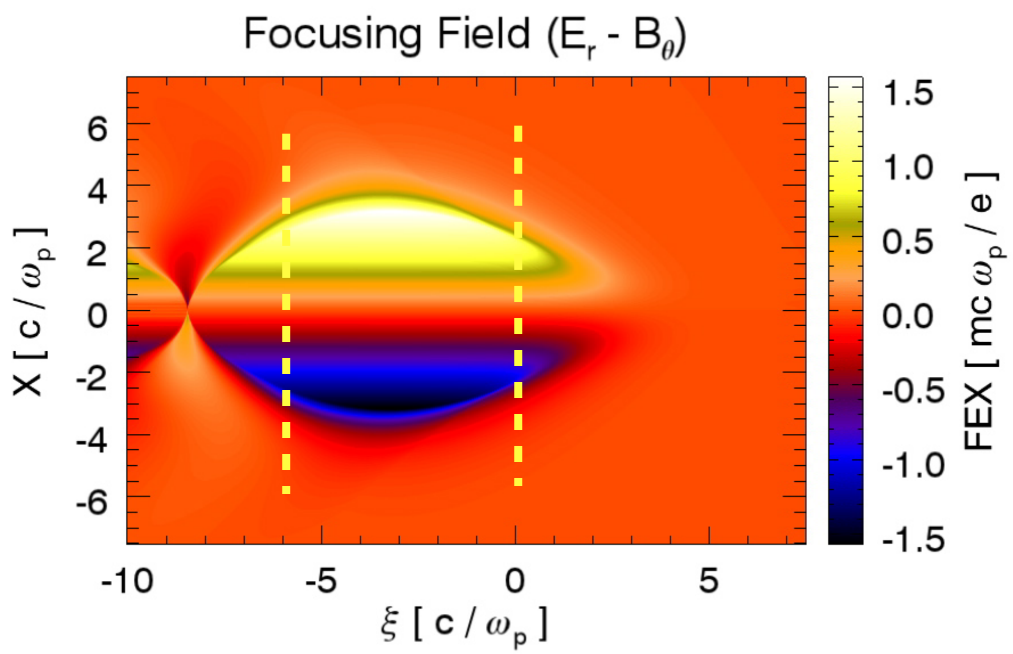
* E. Adli et al., IPAC 2014

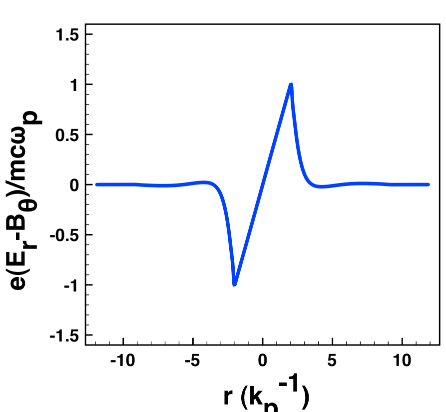
UCLA

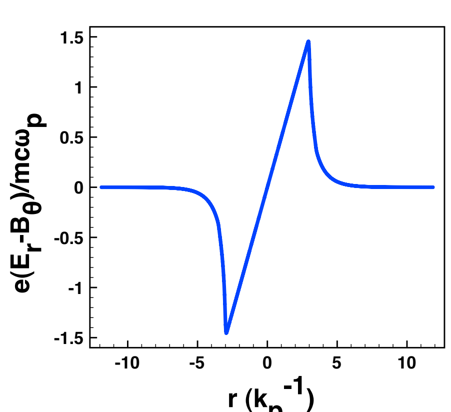
Electron Acceleration in PWFA

Blowout PWFA









Matched spot size for an electron beam

$$\partial f / \partial t = 0$$

$$\sigma_{r,\mathrm{matched}}[\mu m] = 1.304 \left(\frac{\epsilon_N^2 [mm \cdot mrad]}{E[GeV] n_p [10^{16} cm^{-3}]} \right)^{\frac{1}{4}}$$

n_p = 1.0 x 10^{17} cm^{-3}

E = 25 GeV,

ε_N = 0.05 mm·mrad

σ_r = 73.3 nm

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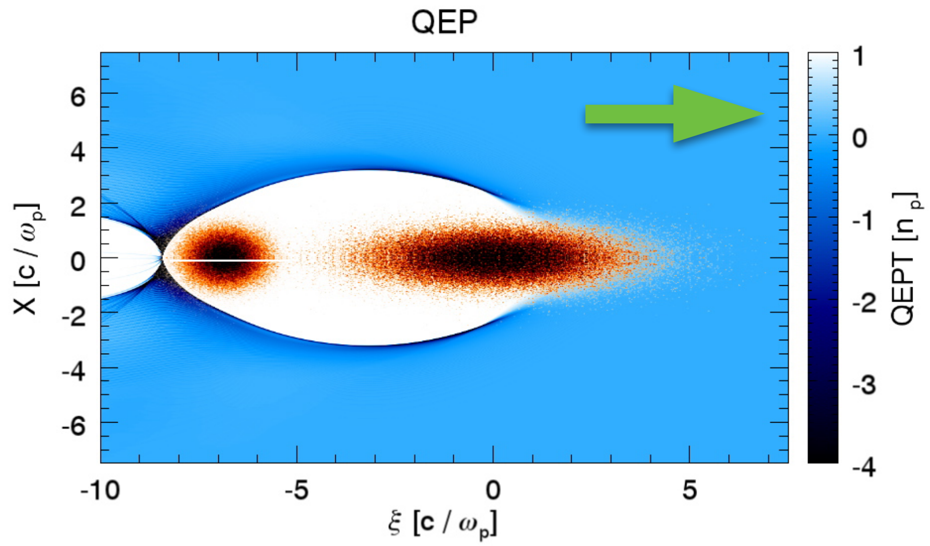
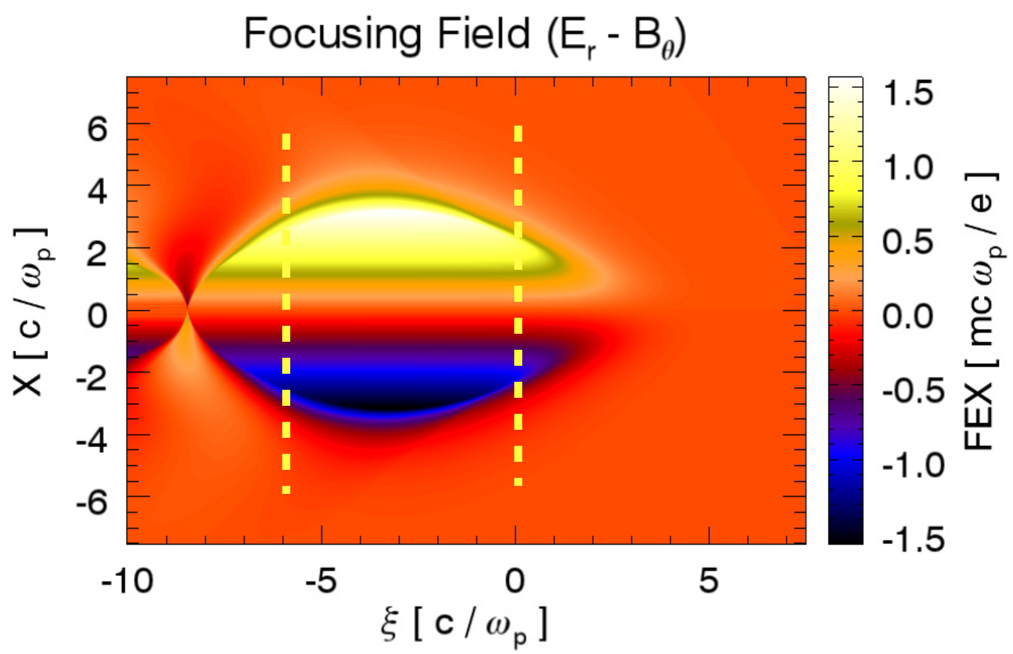
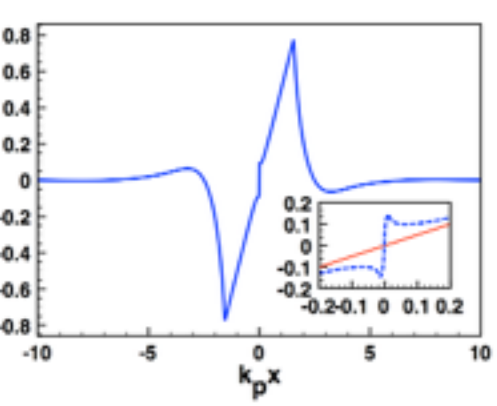
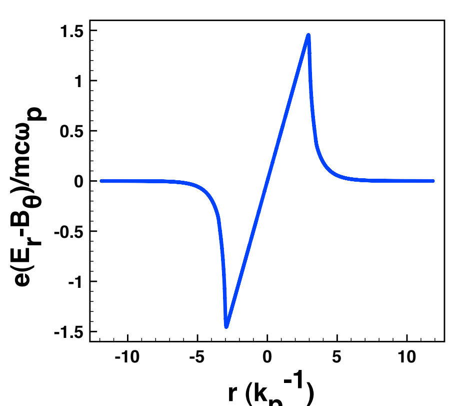
Plasma Ion Motion in PWFA

Blowout PWFA









σ_r = 73.3 nm → Ions collapse!

$$\Omega_b = \sqrt{4\pi n_b e^2/m_i}\ ,\ \Omega_b \sigma_z / c \gg 1$$

*W. An et al. Phys. Rev. Lett. 118, 244801, 2017

$$\sigma_z \approx c/\omega_p\ ,\ \frac{n_b}{n_p} \gg \frac{m_{ion}}{m_e}$$

* J. B. Rosenzweig et al. Phys. Rev. Lett. 95, 195002, 2005

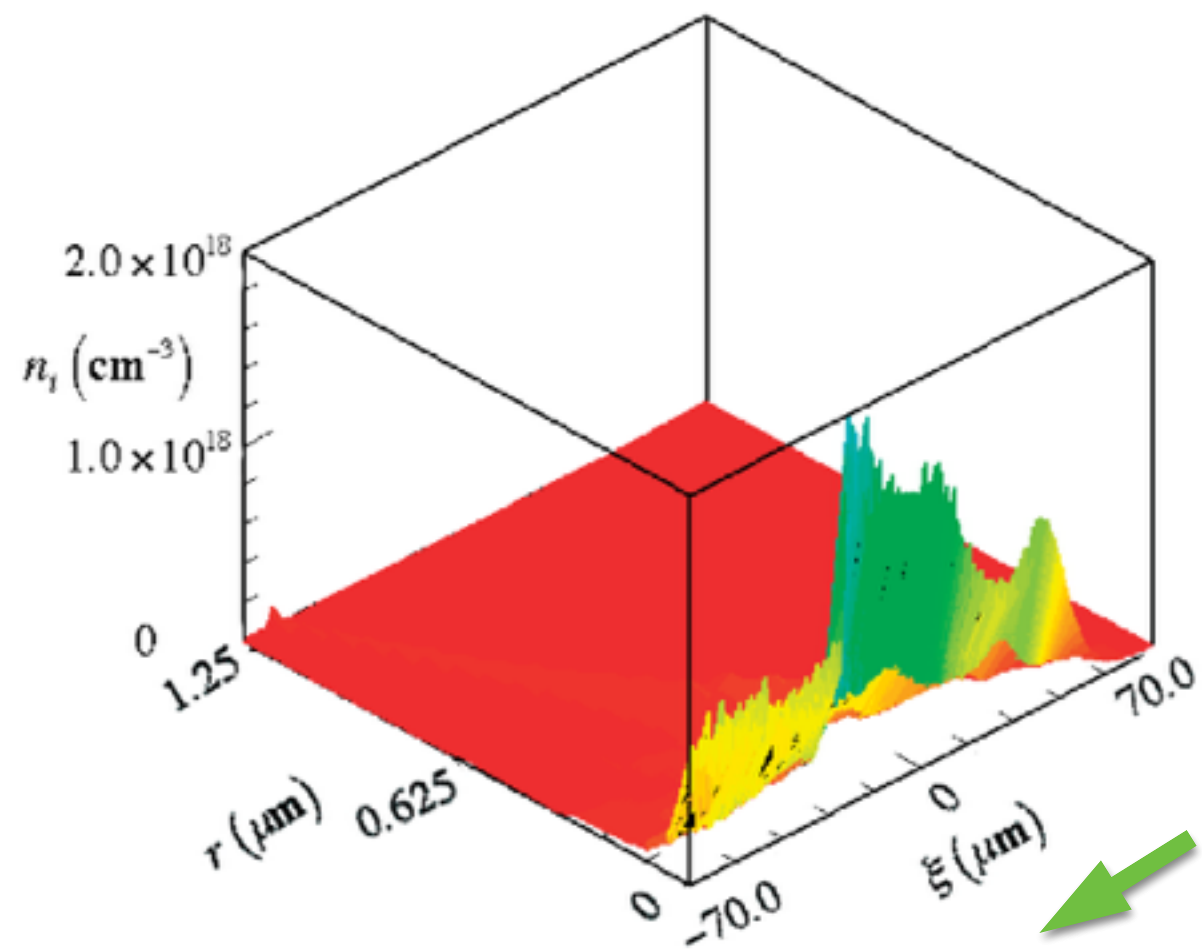
σ_r = 0.1 μm, σ_z = 10.0 μm, N = 1.0 x 10^{10}
n_p = 1.0 x 10^{17} cm^{-3}
n_b/n_p = 63500 >> m_{ion}/m_e = 1836(H) ,12700(Li)

Plasma Ion Motion in PWFA

* J. B. Rosenzweig et al. Phys. Rev. Lett. 95, 195002, 2005

TABLE I. Beam and plasma parameters for linear collider afterburner, derived from Ref. [7].

N_b (drive, accelerating)	1.5×10^{10}, $.5 \times 10^{10}$
rms bunch length σ_z	35 μm
γ (drive, accelerating)	$\leq 1 \times 10^6$, $\leq 2 \times 10^6$
Accelerated beam $\varepsilon_{n,(x,y)}$	4×10^{-6}, 9.6×10^{-6} m rad
Drive beam $\varepsilon_{n,x}$	6.2×10^{-7} m rad
Initial ion (electron) density n_0	0.9×10^{16} cm^{-3}
Ion charge state Z	1 (hydrogen)
Matched β function β_{eq}	3.1 cm
Normalized beam density n_b/n_0	1.5×10^5



n_p = 1.0 x 10^{16} cm^{-3} Hydrogen

FIG. 1 (color). Surface plot of ion density distribution in (ζ, r), as simulated by OOPIC for drive beam conditions of Table I.

[7] Tor O. Raubenheimer, "An Afterburner at the ILC- The Collider Viewpoint", AIPCP 737, 86 (2014)

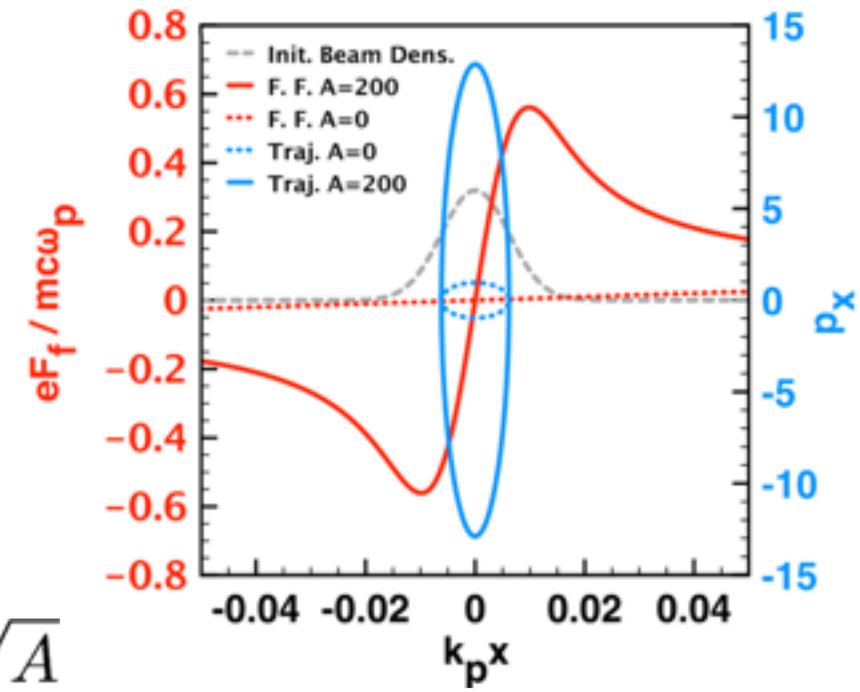
Slice Beam Evolution

The Ion profile: $1 + A\exp(-r^2/2\sigma^2)$

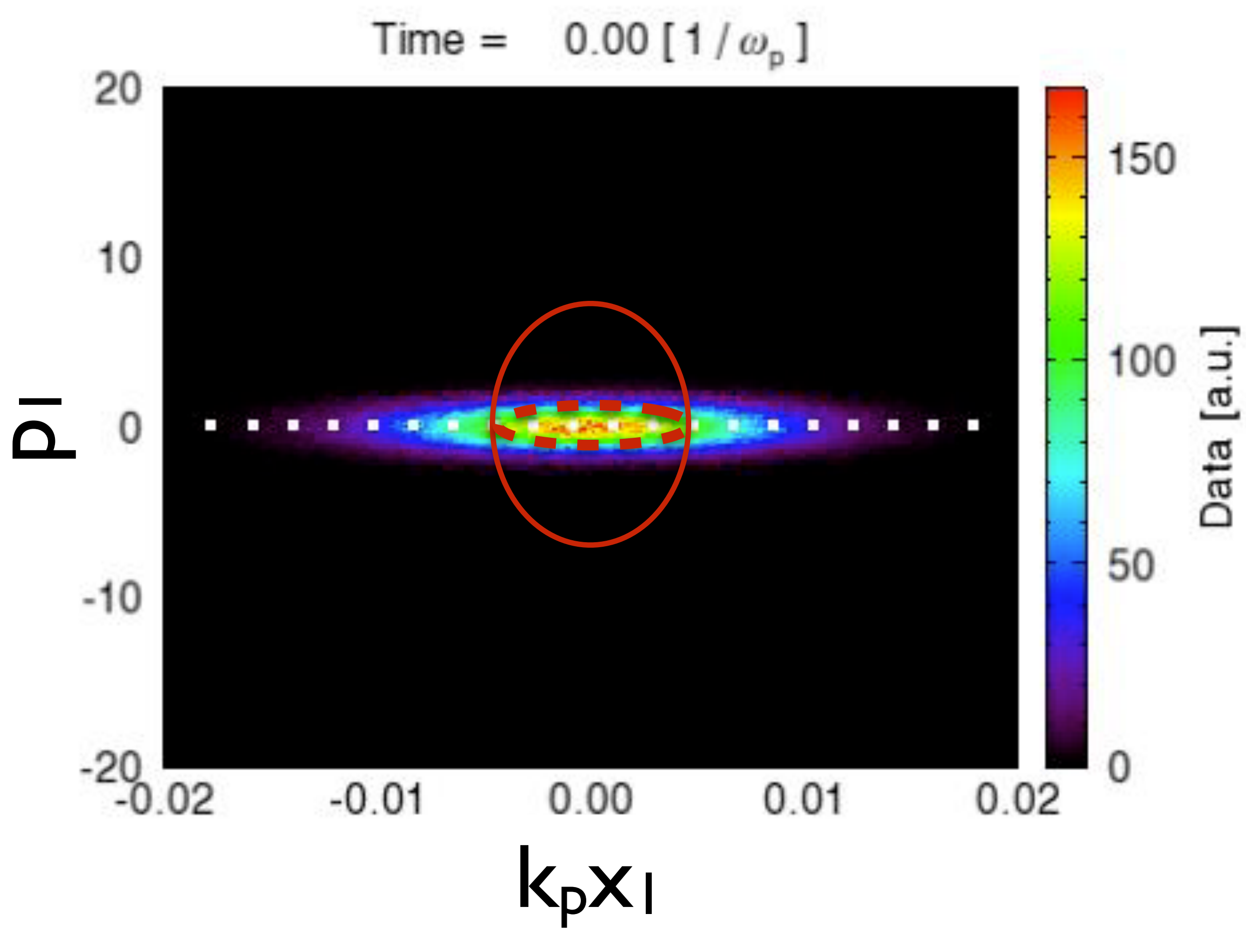
The focusing force: $k_p r/2 + A(k_p\sigma)^2 \frac{1-\exp(-r^2/2\sigma^2)}{k_p r}$

Trailing Beam: σ_r = 0.1 μm , σ_z = 10.0 μm , N = 1.0 x 10^{10}

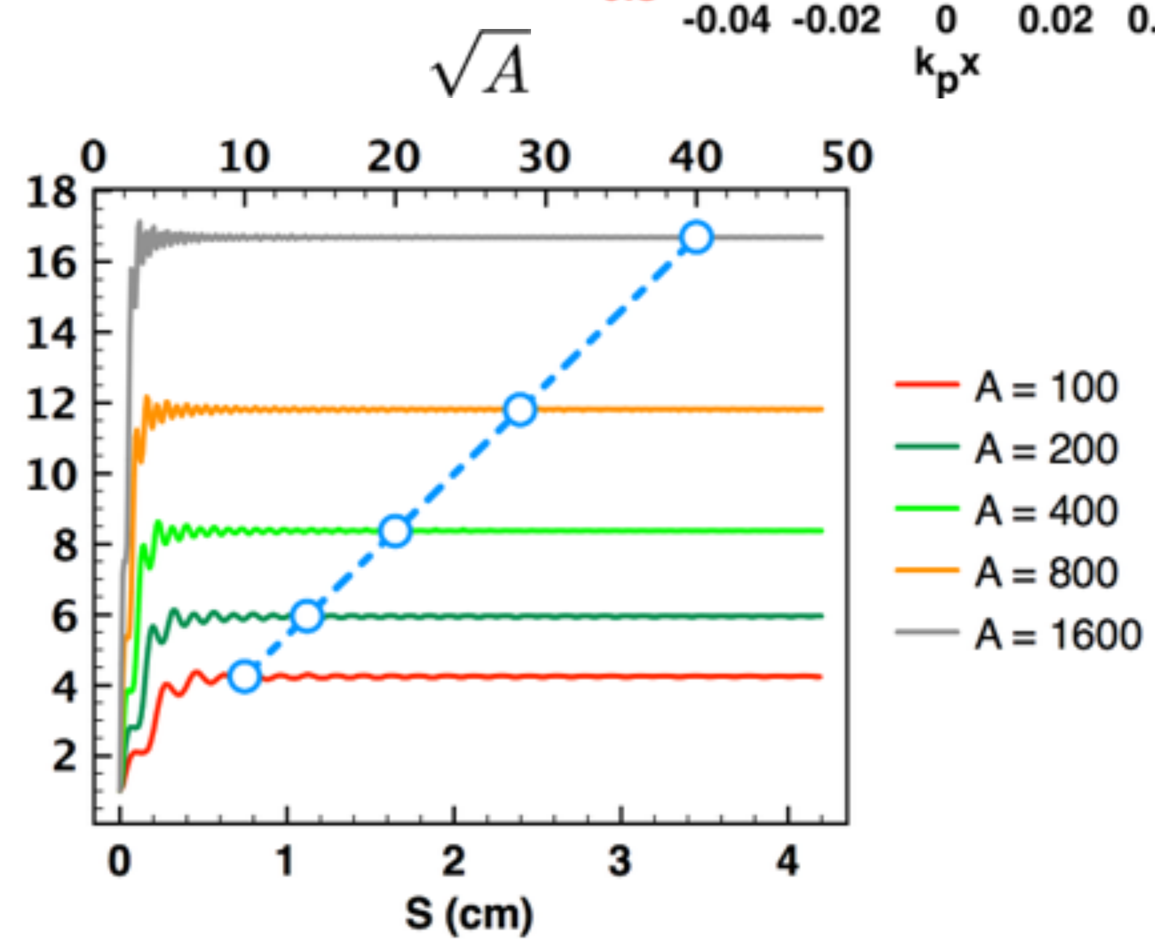
ε_N = **0.093 mm·mrad**



Phase Space of p1x1



Ion Profile: A=100.0, σ = 0.1 μm



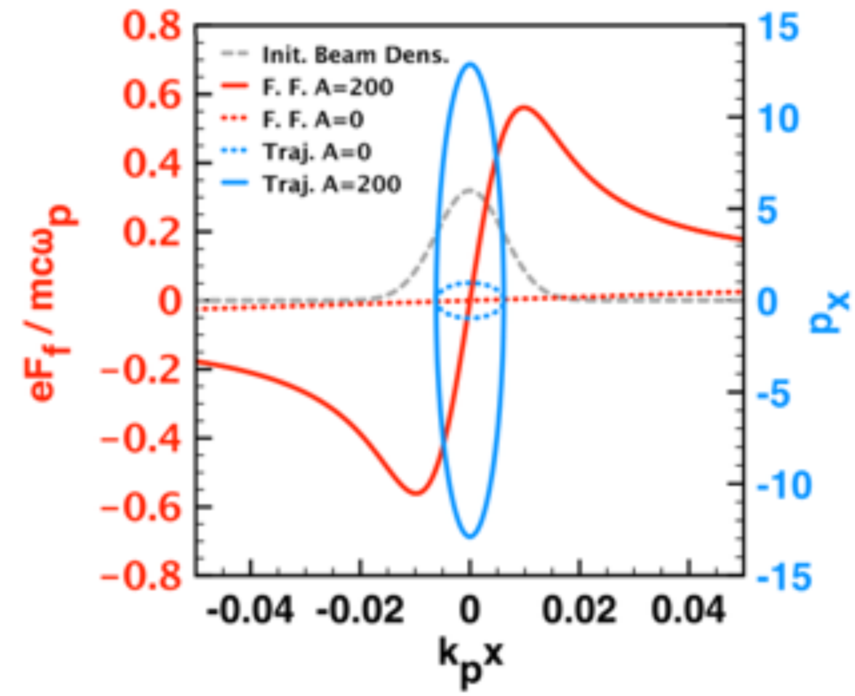
$\varepsilon_{Nf}/\varepsilon_{N0} \propto p_{max} \propto A^{0.5}$

Slice Beam Evolution

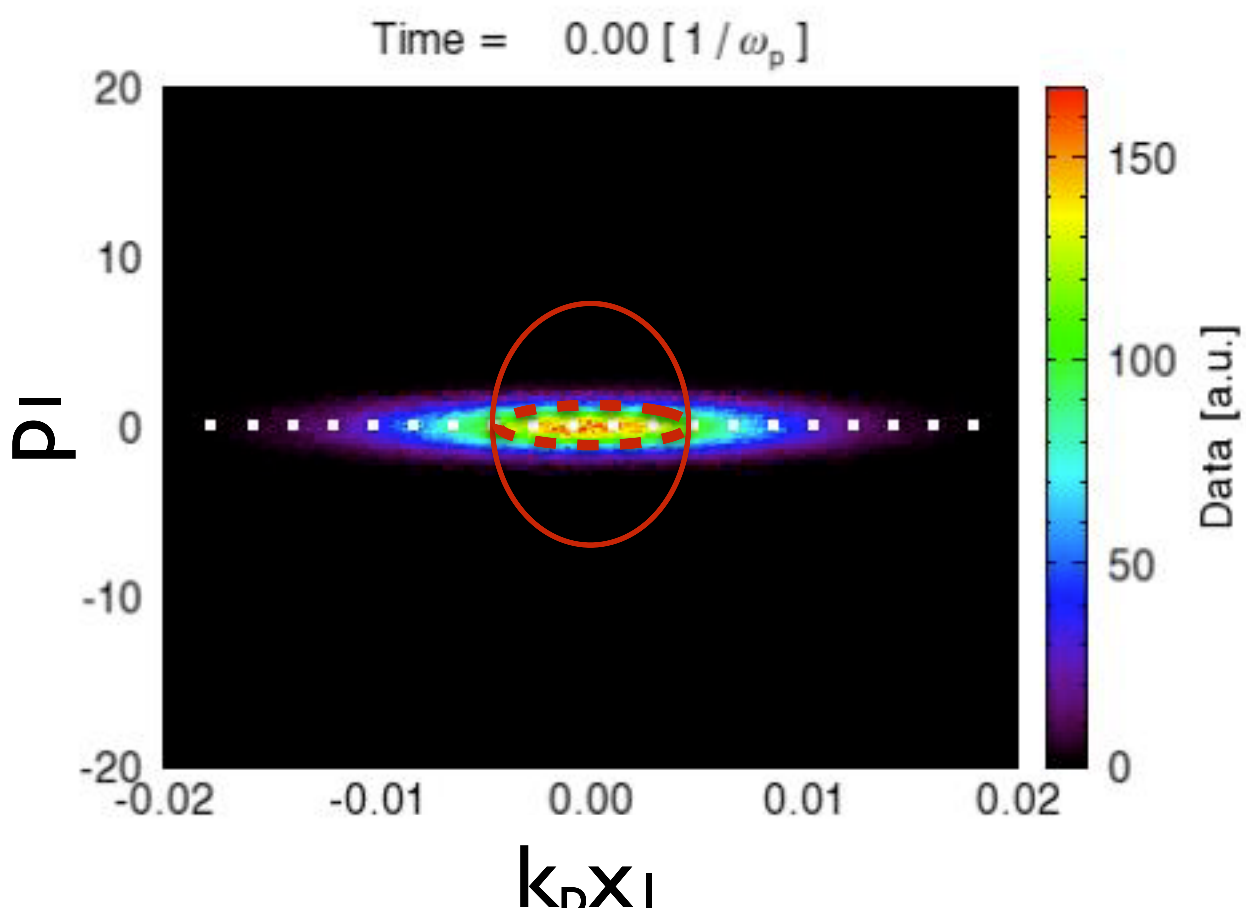
The Ion profile: $1 + A\exp(-r^2/2\sigma^2)$

The focusing force: $k_p r/2 + A(k_p\sigma)^2 \frac{1-\exp(-r^2/2\sigma^2)}{k_p r}$

Trailing Beam: σ_r = 0.1 μm , σ_z = 10.0 μm , N = 1.0 x 10^{10}
ε_N = **0.093 mm·mrad**



Phase Space of p1x1



Ion Profile: A=100.0, σ = 0.1 μm

$$\epsilon_N = \gamma\sqrt{<x^2><x'^2> - <xx'>^2}$$

$$\epsilon'_N = \frac{\gamma}{2\epsilon_N}(<x^2><x'x''> - <xx'><xx''>)$$

$$\text{If} \quad x'' = cx, \epsilon'_N = 0$$

Plasma Ion Motion in PWFA

* J. B. Rosenzweig et al. Phys. Rev. Lett. 95, 195002, 2005

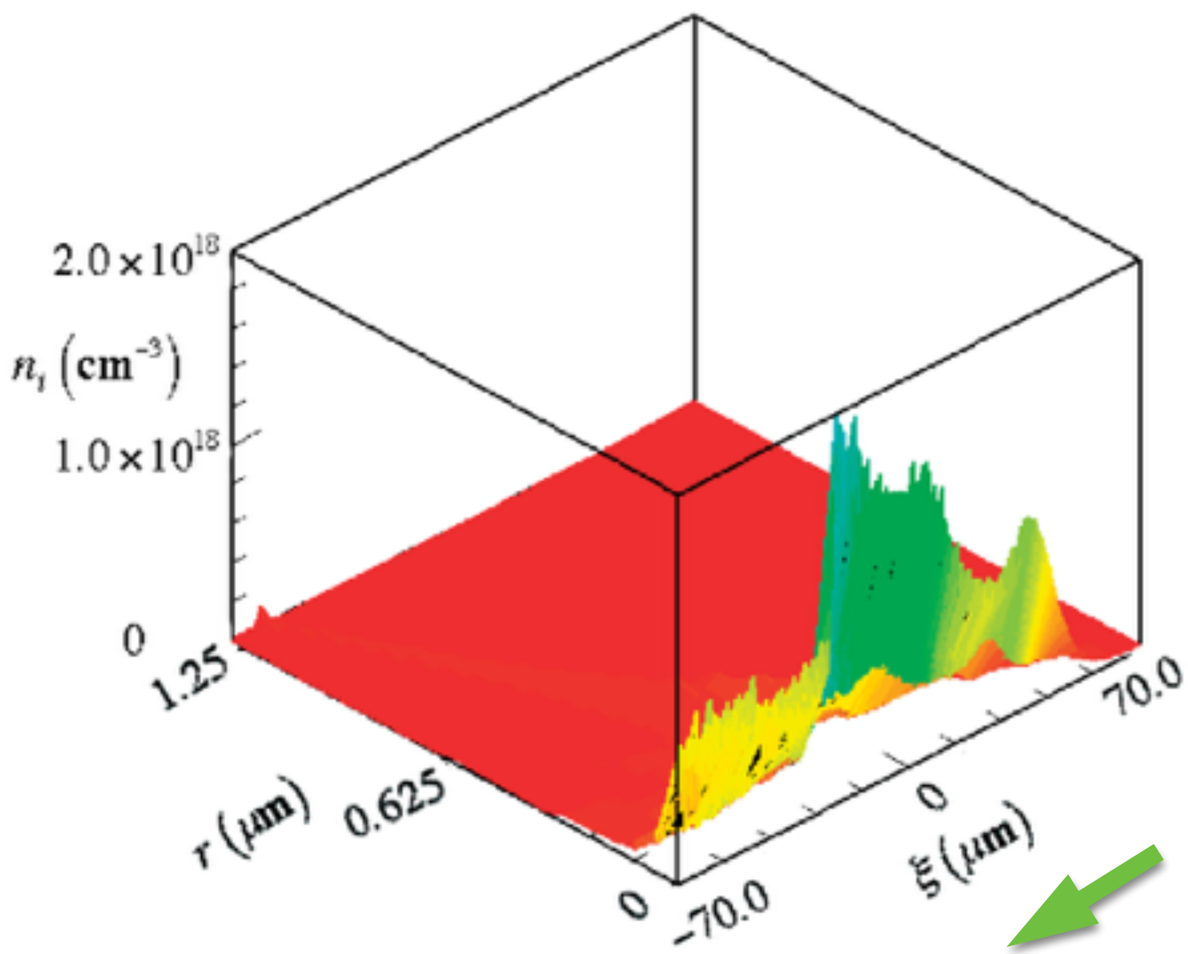
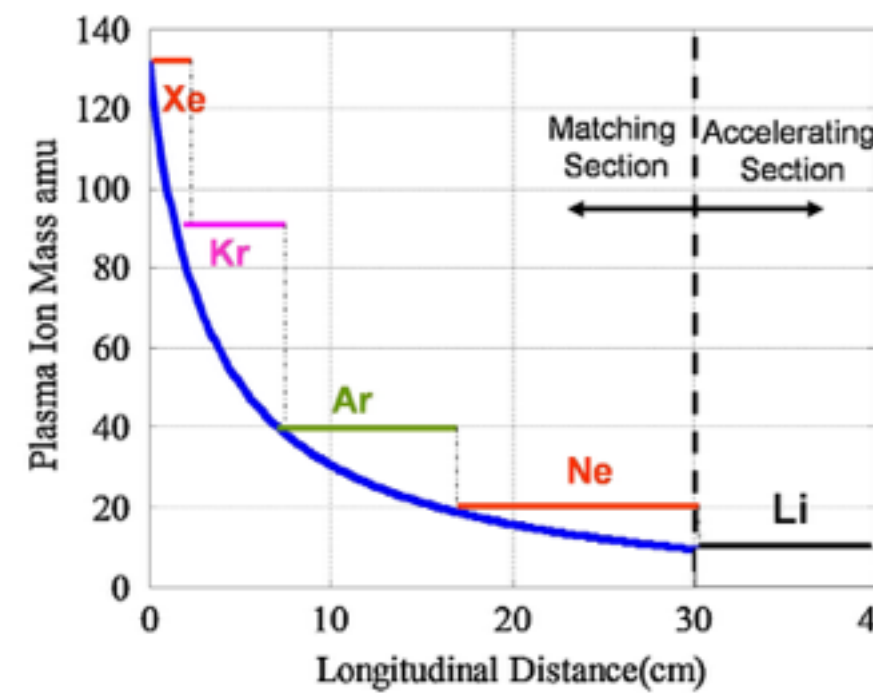
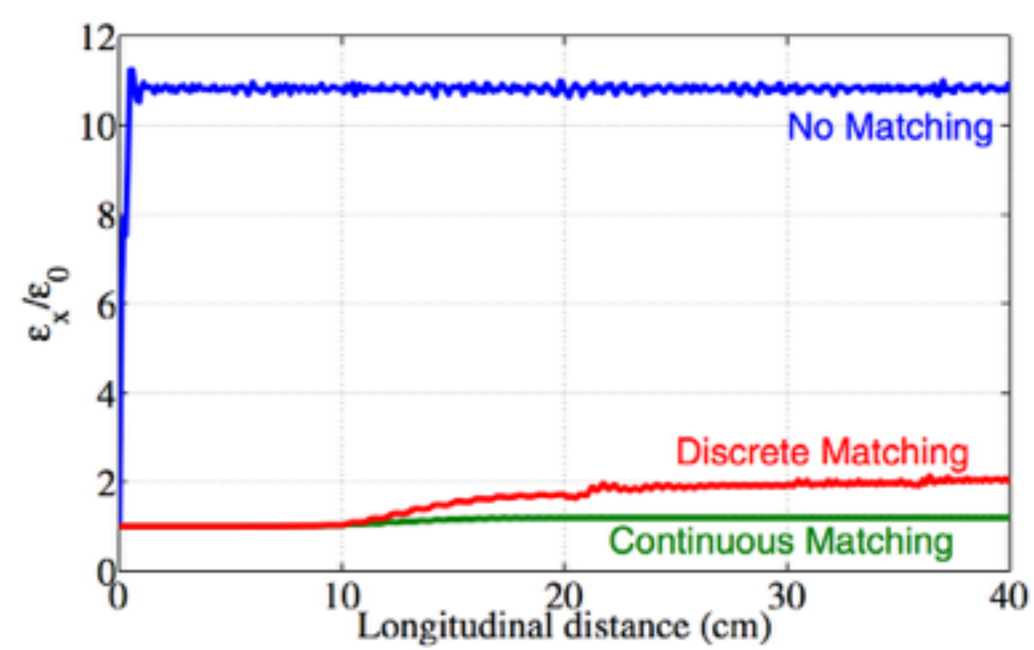


FIG. 1 (color). Surface plot of ion density distribution in (ζ, r), as simulated by OOPIC for drive beam conditions of Table I.

$$d\varepsilon_{n,x}/dz \simeq 6\times 10^{-4}\ \mathrm{m\,rad/m}$$

* R. Gholizadeh et al. Phys. Rev. Lett. 104, 155001, 2010

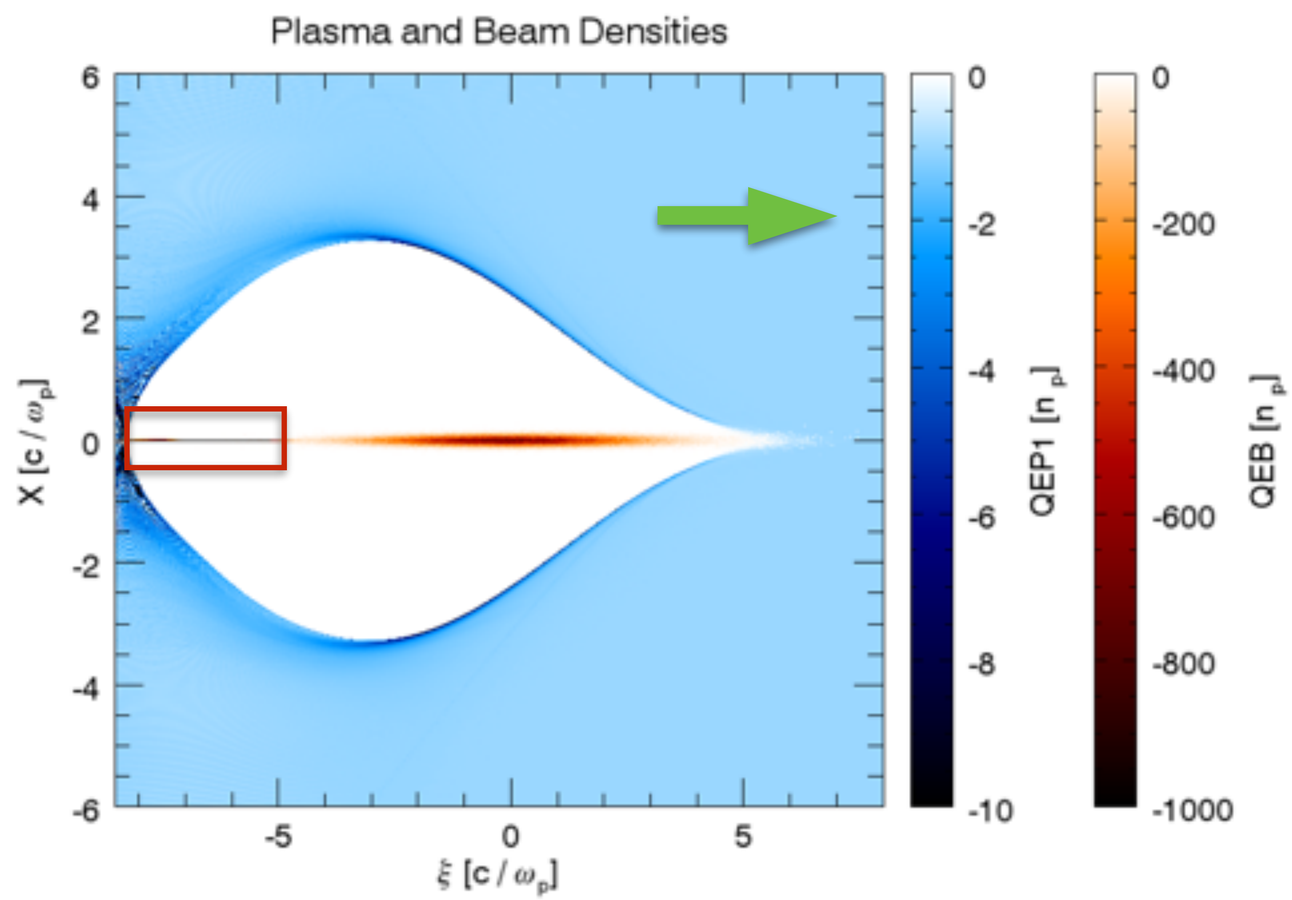


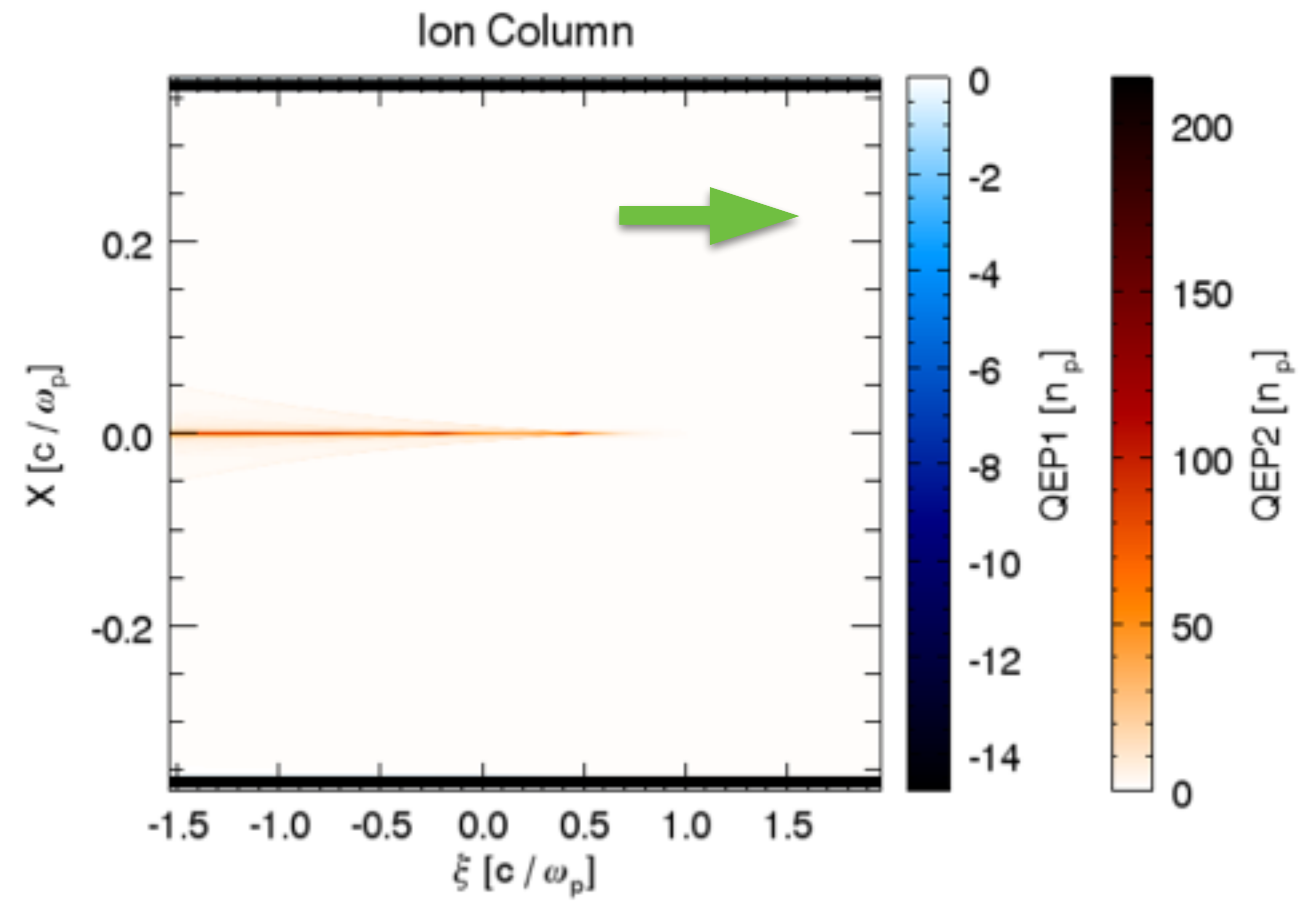


Simulation Using QuickPIC

3D Simulation Big Challenge

σ_r = 0.1 μm, σ_z = 10.0 μm, N = 1.0 x 10^{10}
n_b/n_p = 63500 >> m_{ion}/m_e = 1836





400 μm x 400 μm x 300 μm Box

8192 x 8192 x 1024 Cells

$\Delta_\perp \approx$ 50 nm

12 μm x 12 μm x 60 μm Box

4096 x 4096 x 512 Cells

$\Delta_\perp \approx$ 3 nm

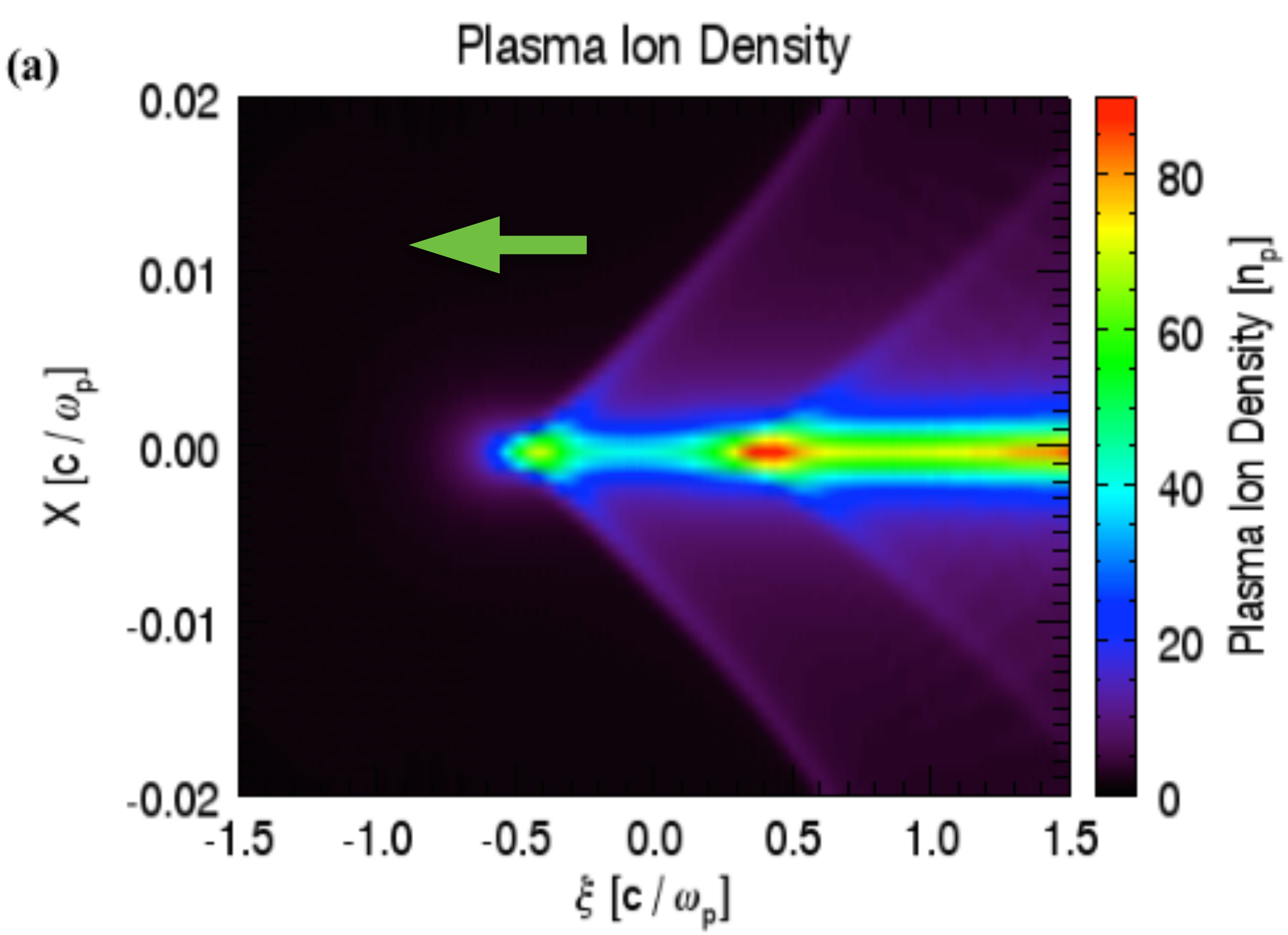
UCLA

Simulation Using QuickPIC

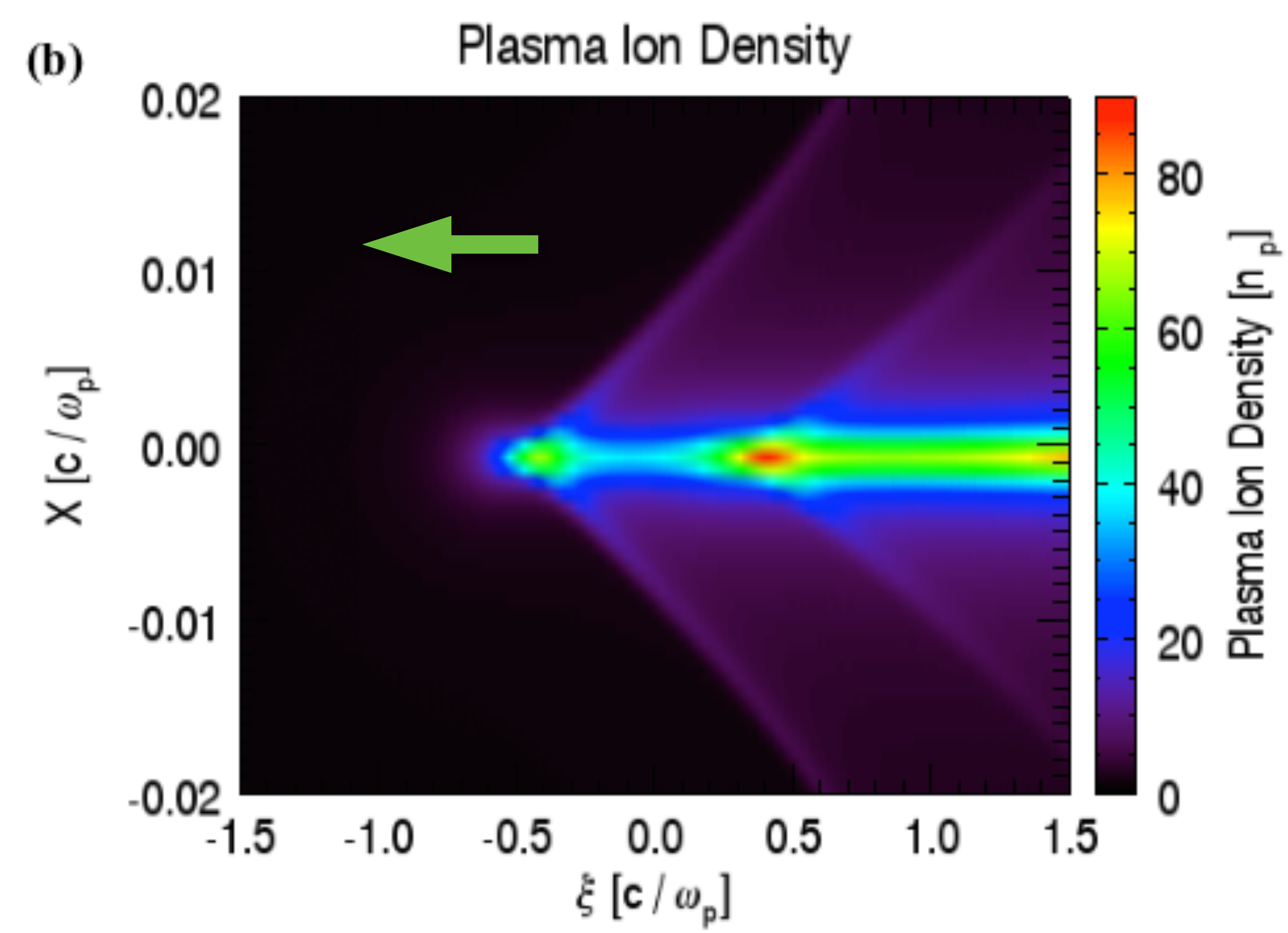
400 µm x 400 µm x 313 µm Box | **12 µm x 12 µm x 60 µm Box**

$\Delta_\perp \approx 12$ nm

32768 x 32768 x 2693 Cells | 1024 x 1024 x 512 Cells



Two-Bunch

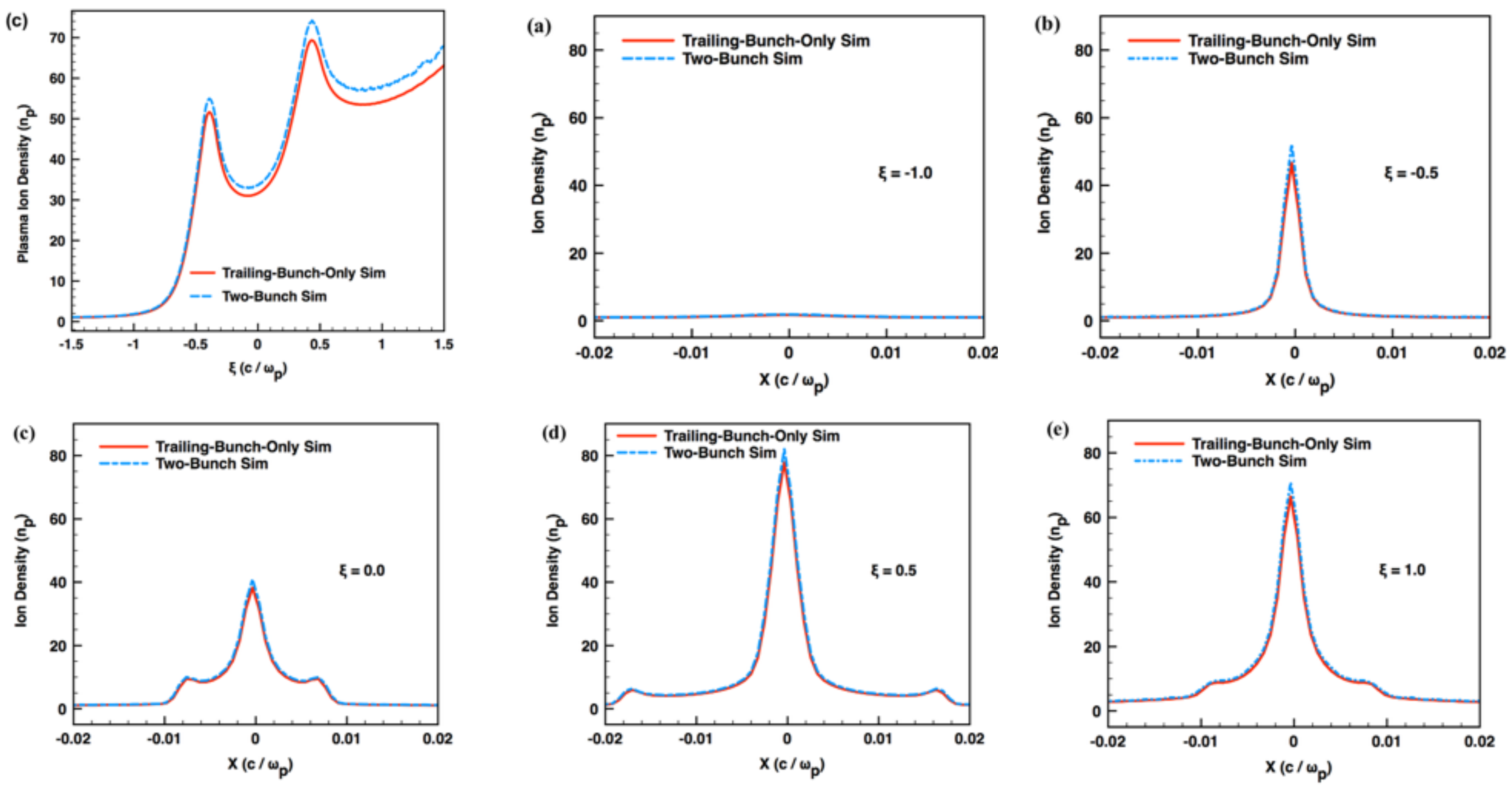


Trailing-Bunch-Only

UCLA

Simulation Using QuickPIC

Comparison of Longituidinal and Transverse Lineouts



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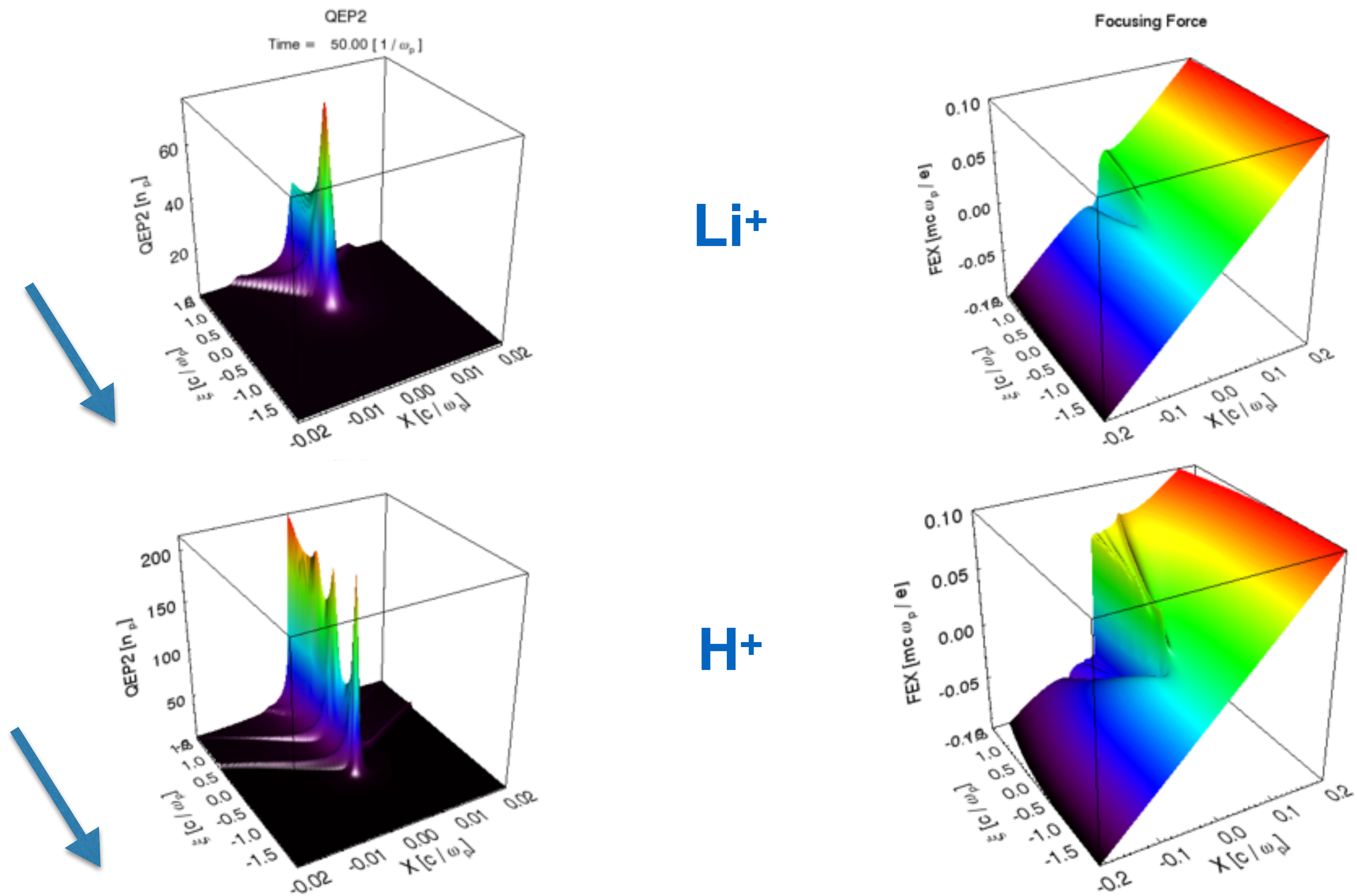
Ion Motion in PWFA

Drive Beam : σ_r = 1 μm , σ_z = 30.0 μm , N_1 = 3.0 x 10^{10} , ε = **10 mm·mrad**

Trailing Beam: σ_r = 0.1 μm (0.006 k_p^{-1}), σ_z = 10.0 μm, N_2 = 1.0 x 10^{10}, ε = **0.093 mm·mrad**

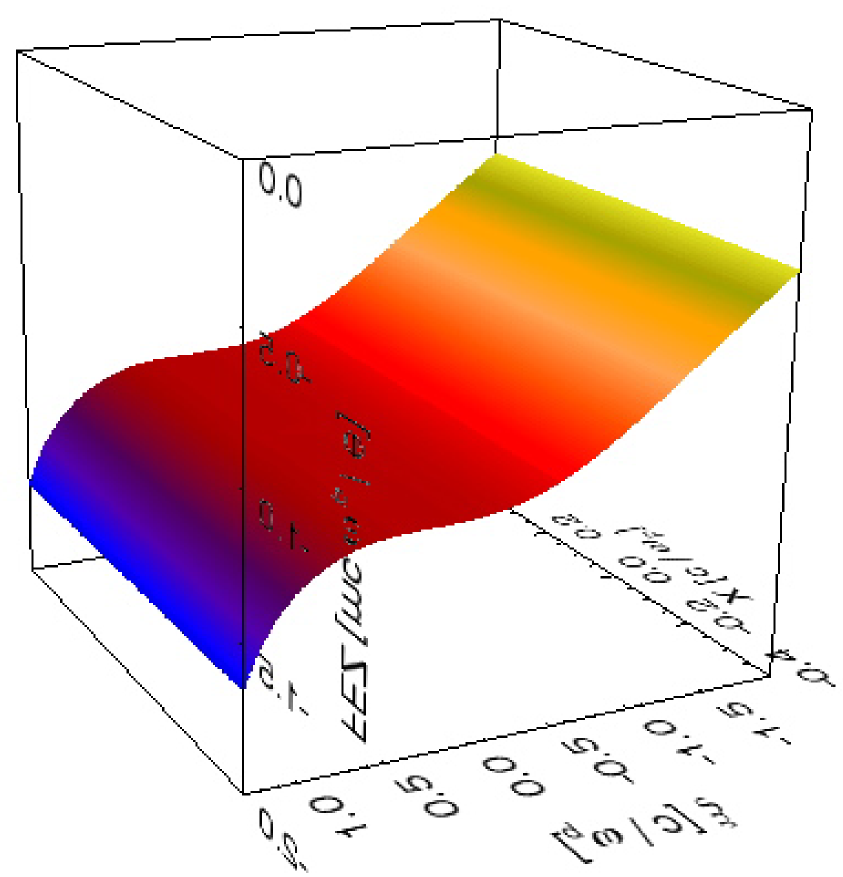
Distance between two beams : 115 μm; Plasma Density : 1.0 x 10^{17} cm^{-3}

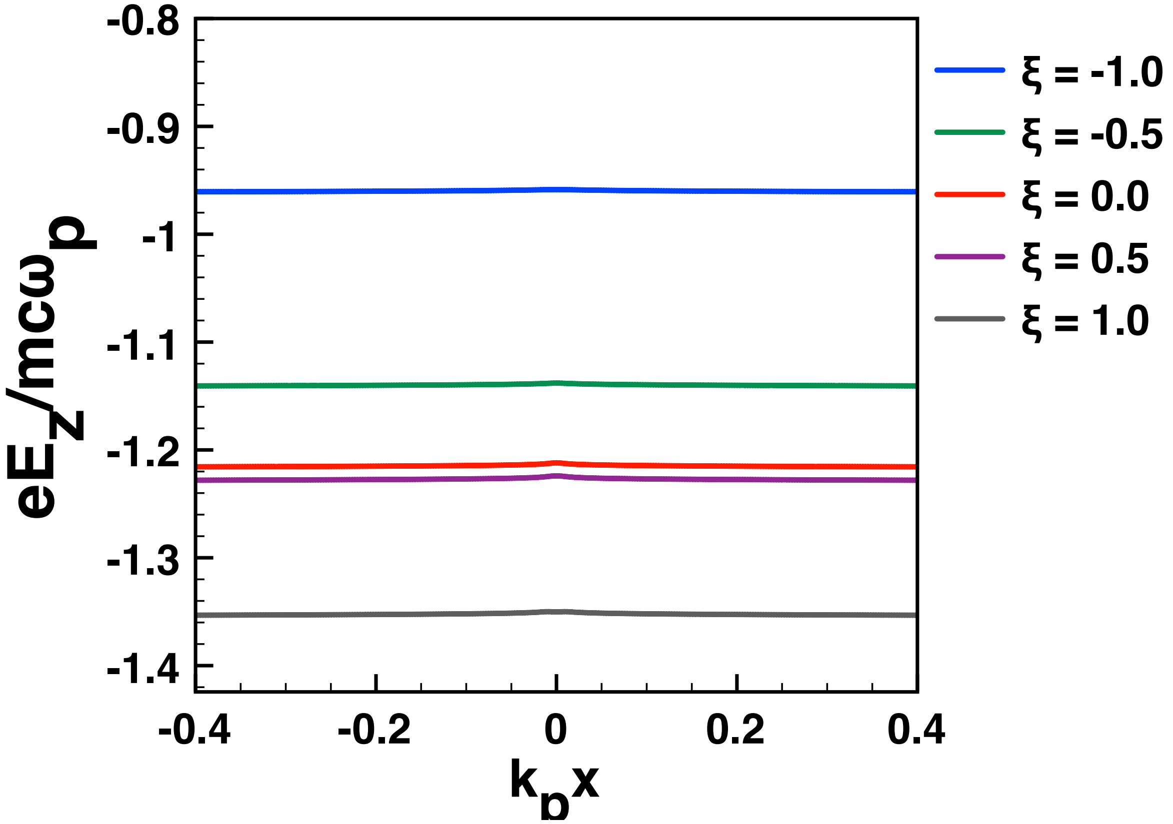


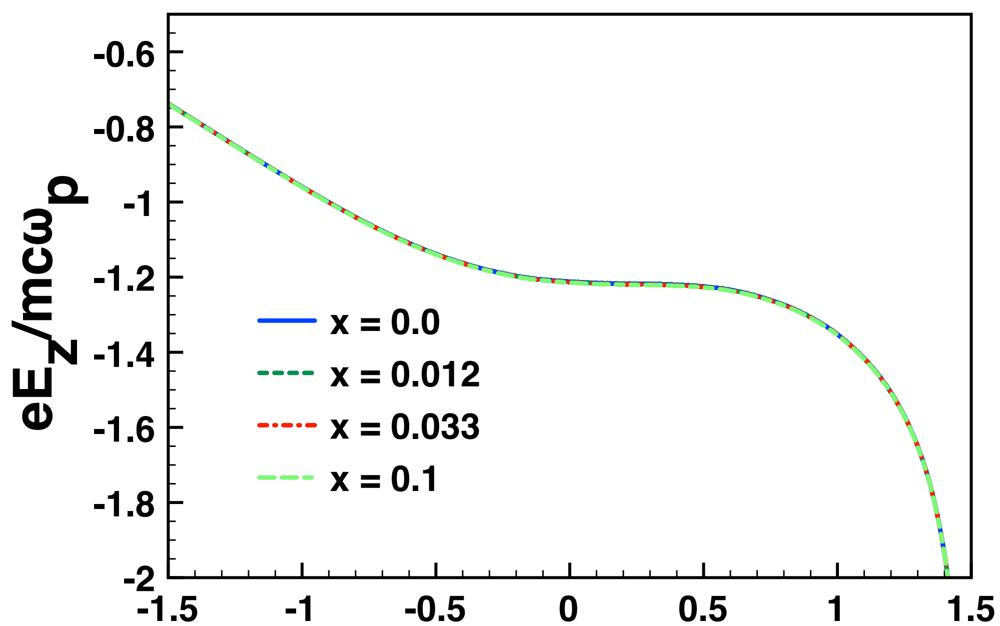
Plasma Ion Motion in a PWFA-LC Stage

The accelerating field of the plasma wake

Drive Beam : σ_r = 1 μm , σ_z = 30.0 μm , $N_1 = 3.0 \times 10^{10}$, ε = **10 mm·mrad**
Trailing Beam: σ_r = 0.1 μm, σ_z = 10.0 μm, $N_2 = 1.0 \times 10^{10}$, ε = **0.093 mm·mrad**
Distance between two beams : 115 μm;
Plasma Density : 1.0×10^{17} cm^{-3} (Field Ionized Li Plasma)







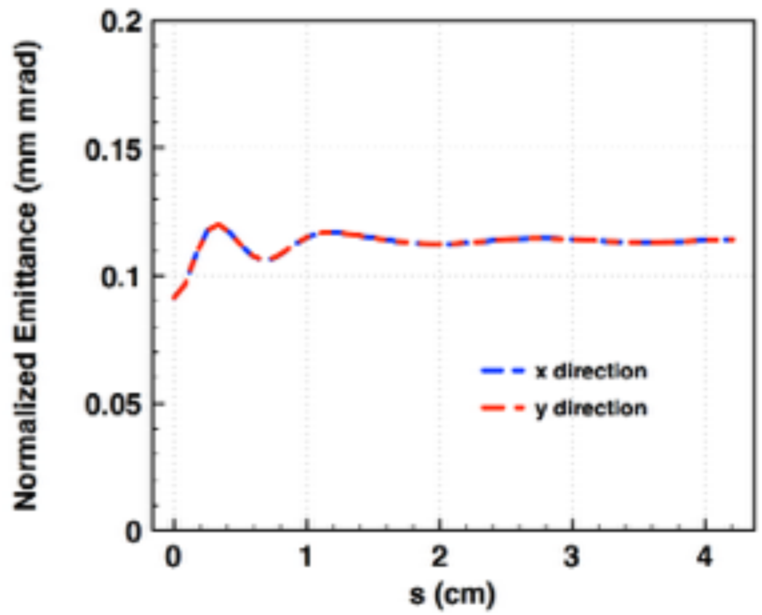
*W. An et al. Phys. Rev. Lett.118, 244801, 2017

$$\Delta E_z = \int dr \partial F_f / \partial \xi \approx \Delta r \Delta F_f / 2\Delta \xi$$

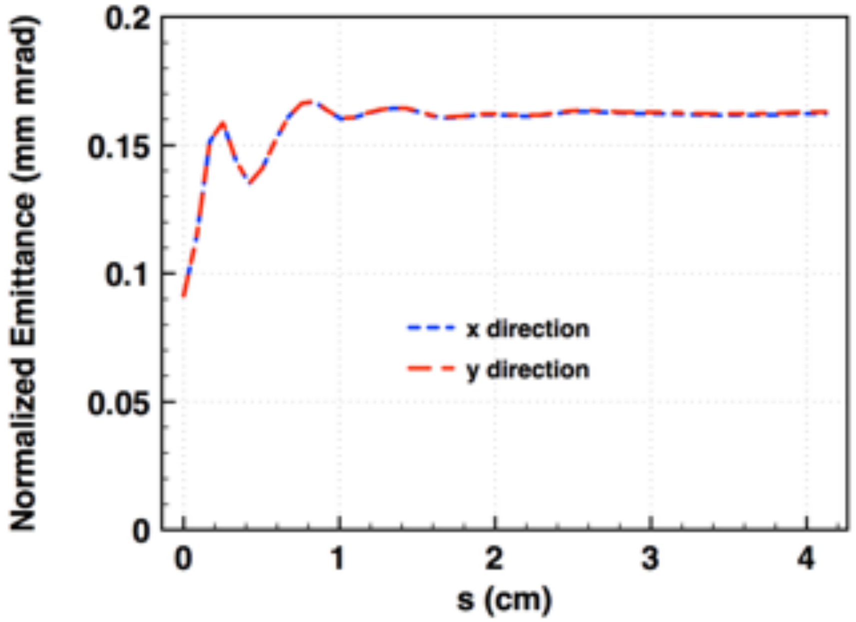
Ion Motion in PWFA

The Evolution of the electron beam's emittance

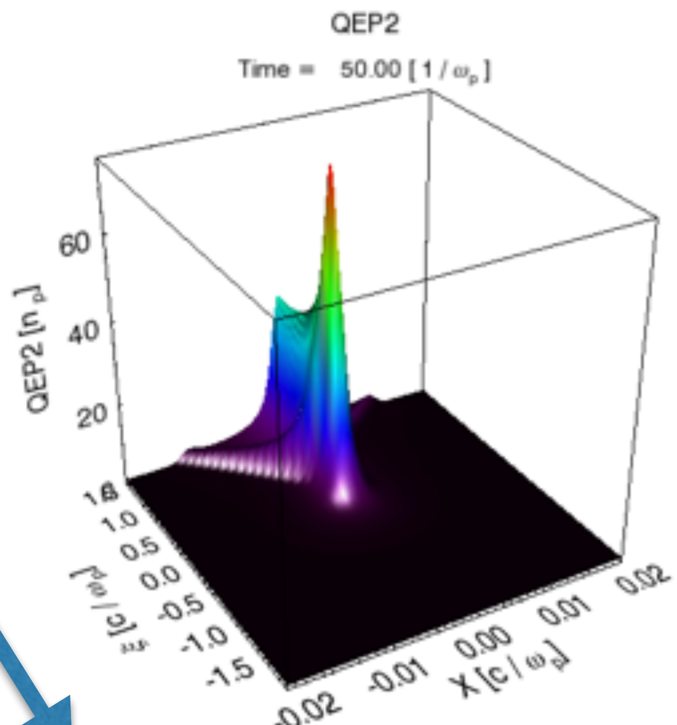
Li+

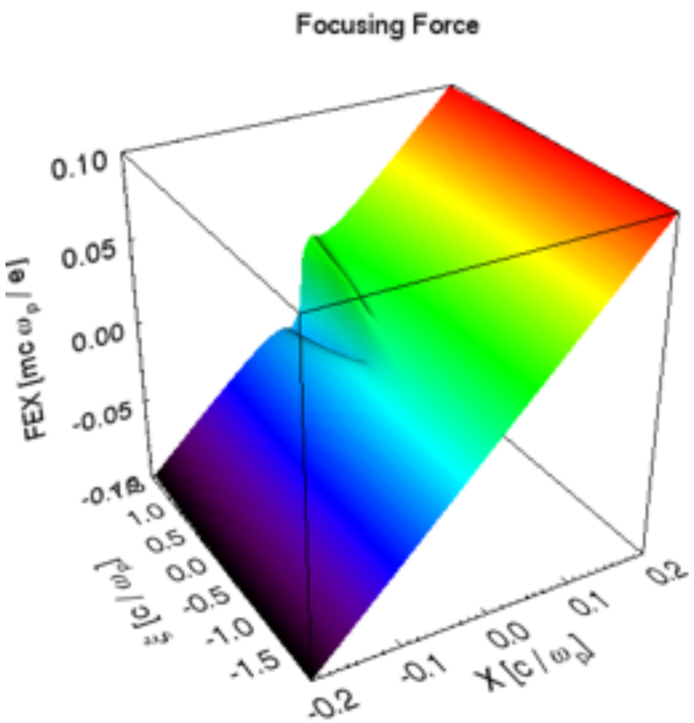


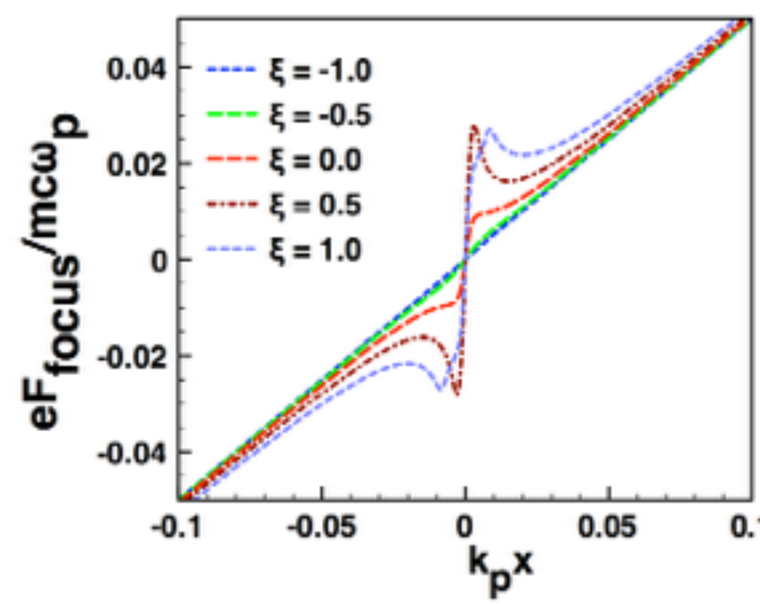
H+



Li+







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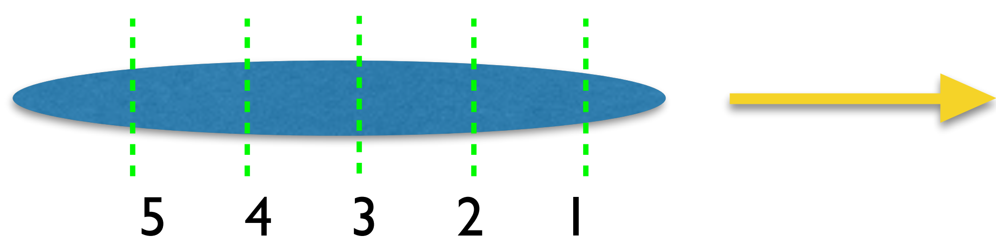
Ion Motion in PWFA

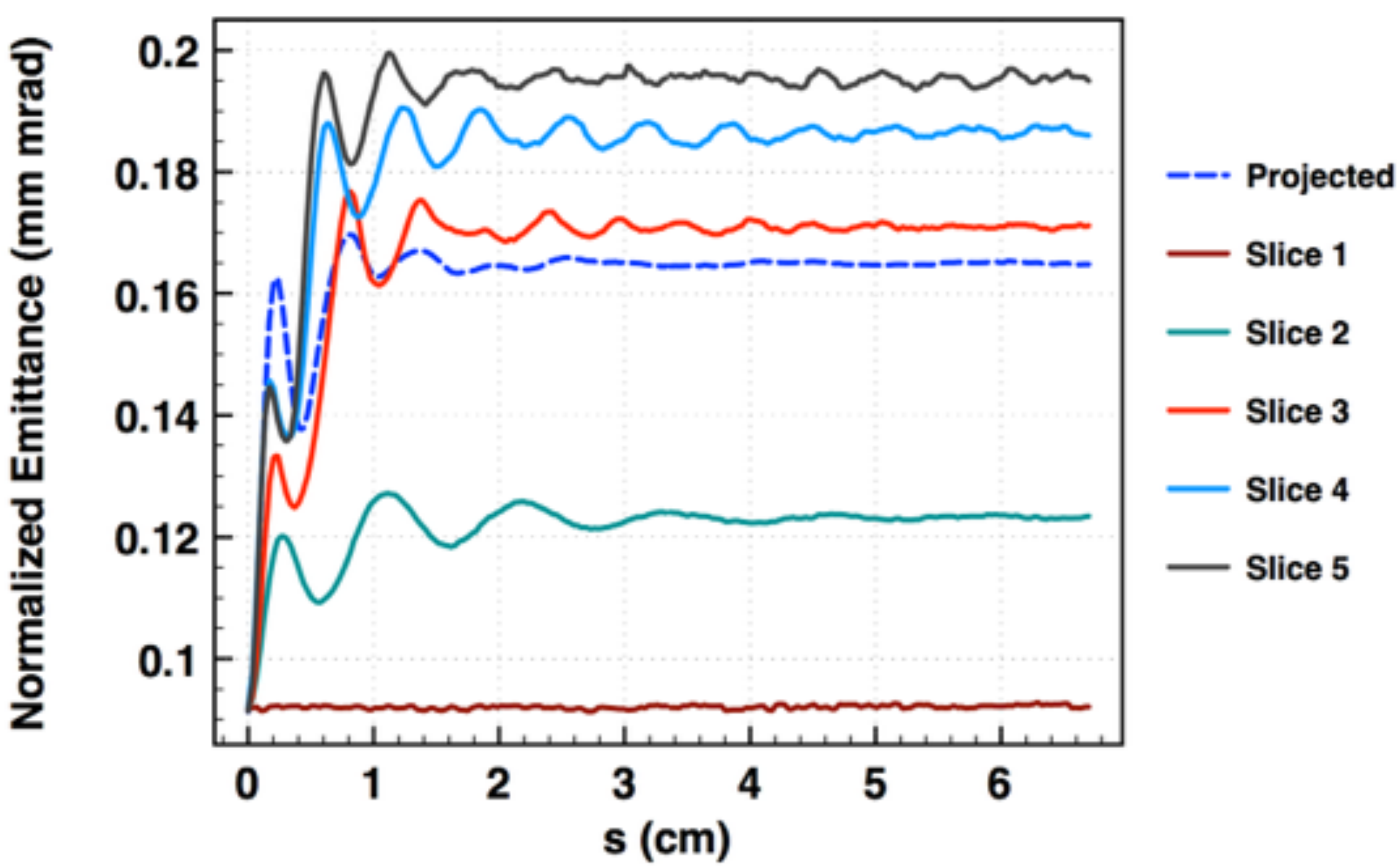
Emittance growth of the trailing beam

Drive Beam : σ_r = 1 μm , σ_z = 30.0 μm , N_1 = 3.0 x 10^{10}, ε = **10 mm·mrad**

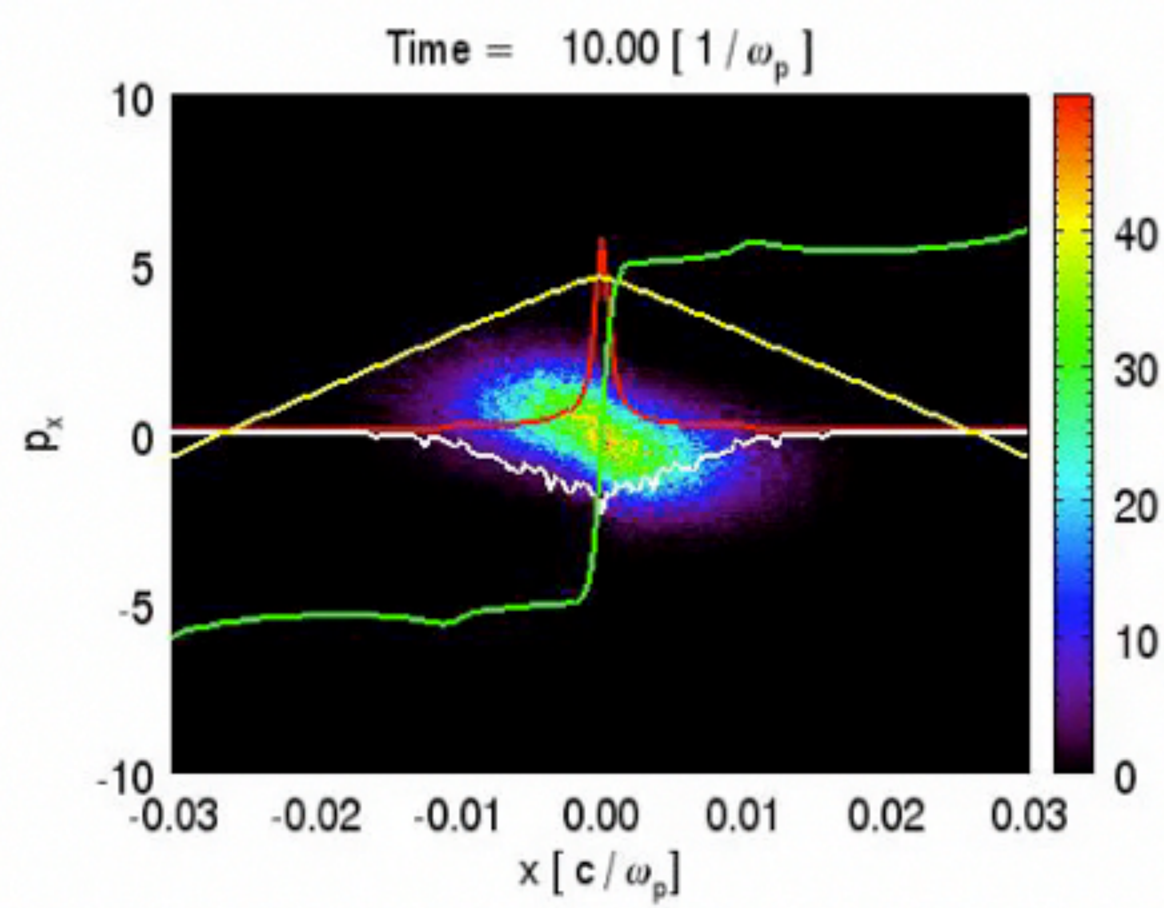
Trailing Beam: σ_r = 0.1 μm (0.006 k_p^{-1}) , σ_z = 10.0 μm , N_2 = 1.0 x 10^{10} , ε = **0.093 mm·mrad**

Distance between two beams : 115 μm Plasma Density : 1.0 x 10^{17} cm^{-3} (H Plasma)





Slice 5



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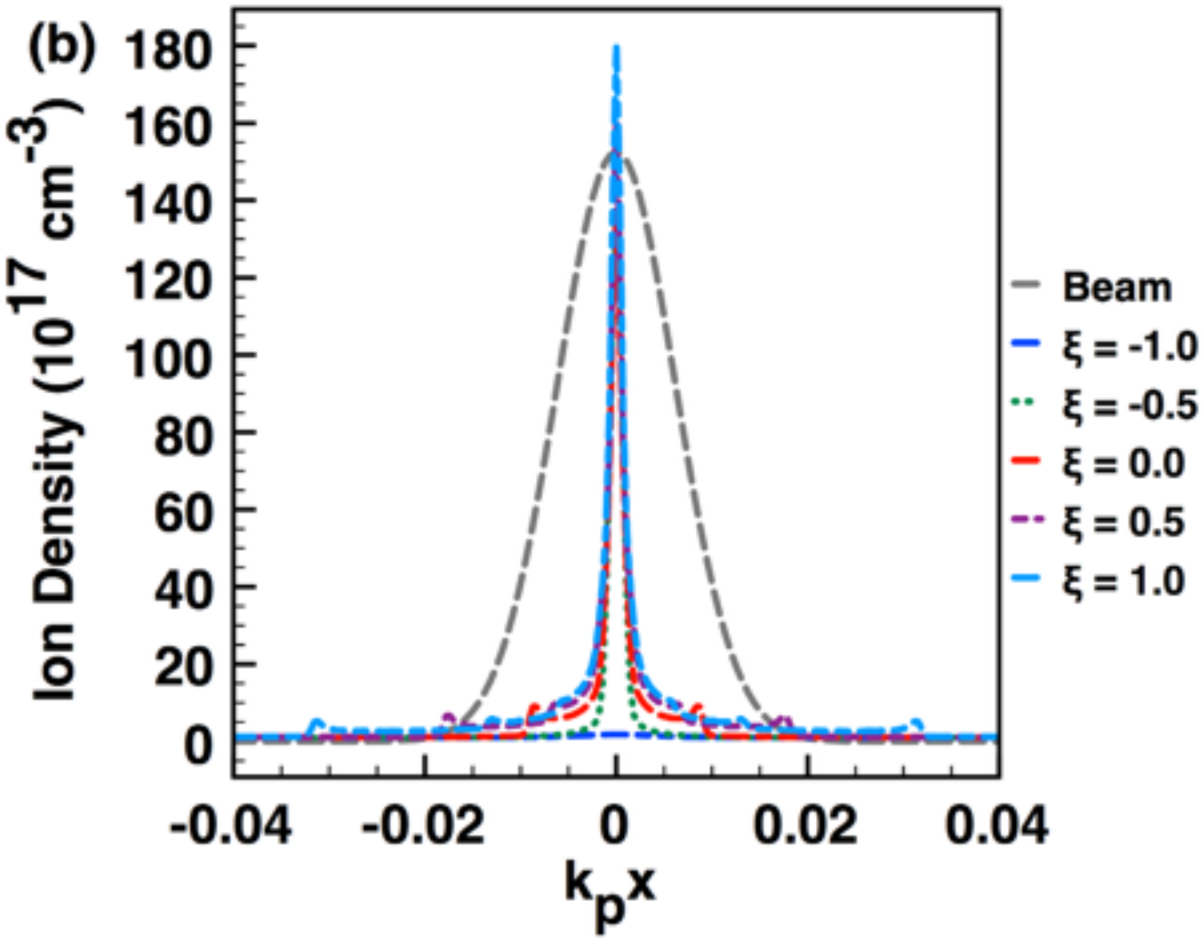
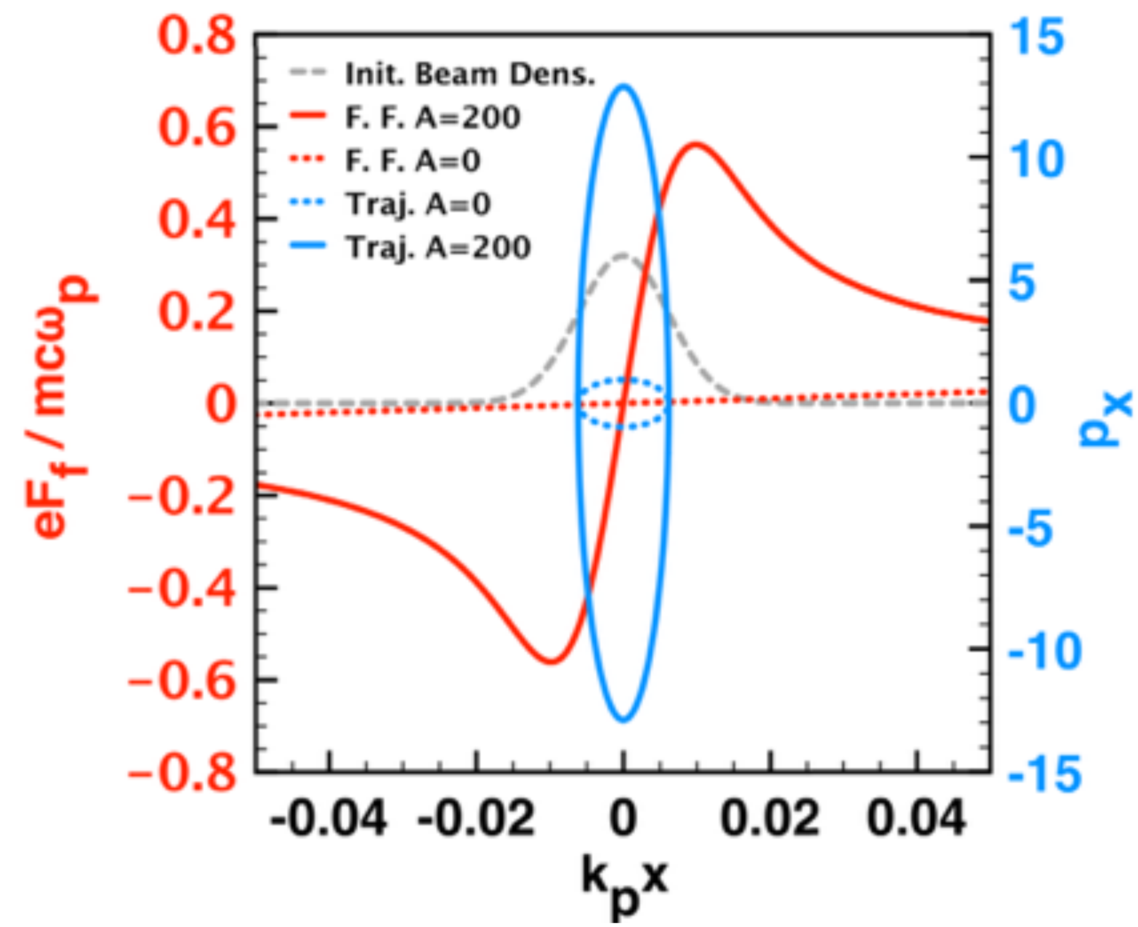
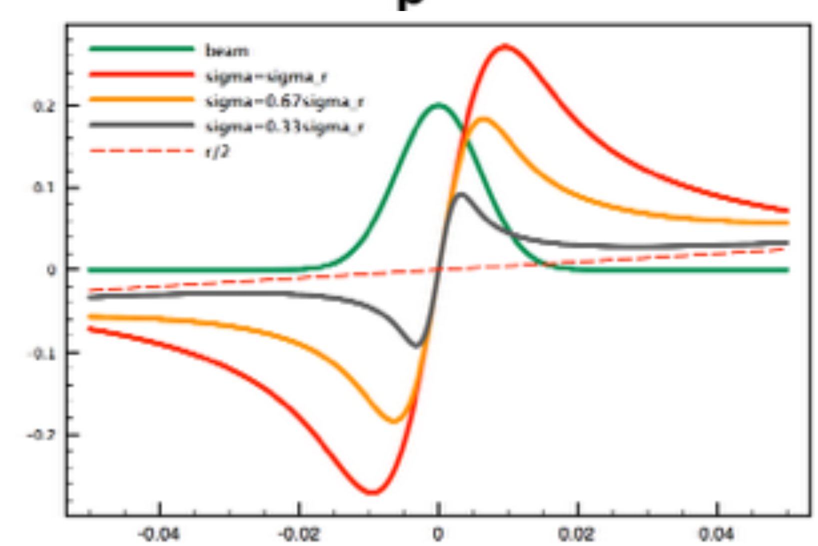
Ion Motion in PWFA

Ion density peak

The Ion profile: $1 + A\exp(-r^2/2\sigma^2)$







$\varepsilon_{Nf}/\varepsilon_{N0} \propto p_{max} \propto$ ~~$A^{0.5}$~~

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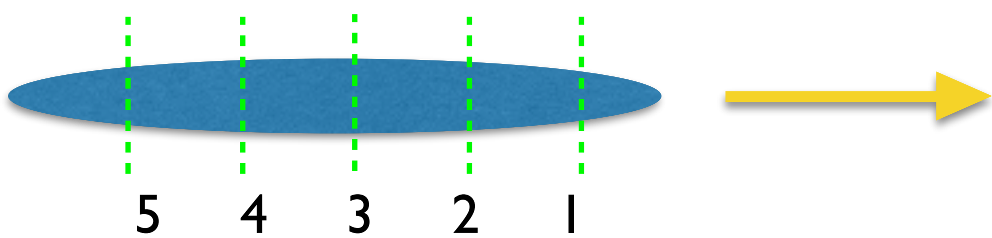
Ion Motion in PWFA

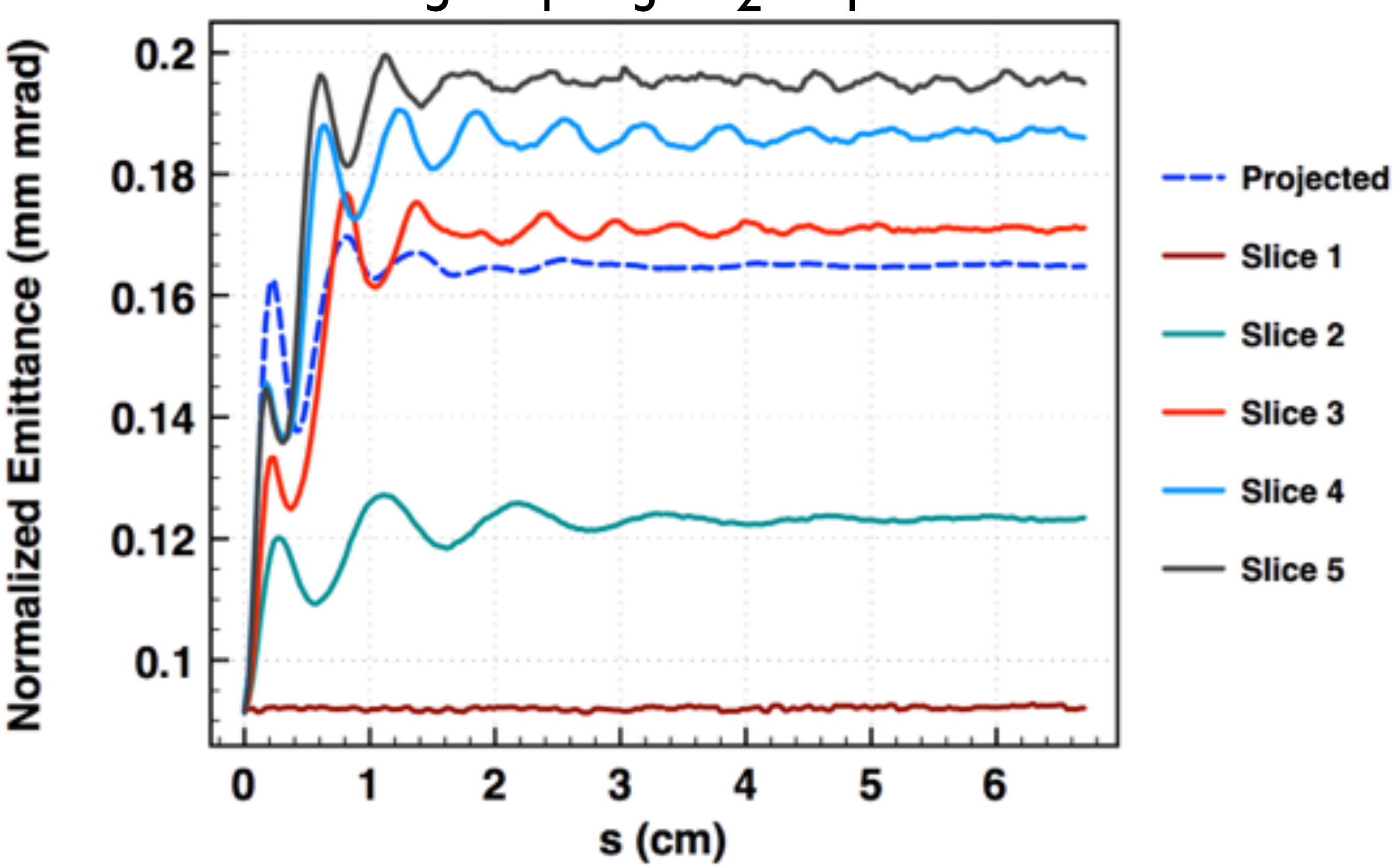
Emittance growth of the trailing beam

Drive Beam : σ_r = 1 μm , σ_z = 30.0 μm , $N_1 = 3.0 \times 10^{10}$, ε = **10 mm·mrad**

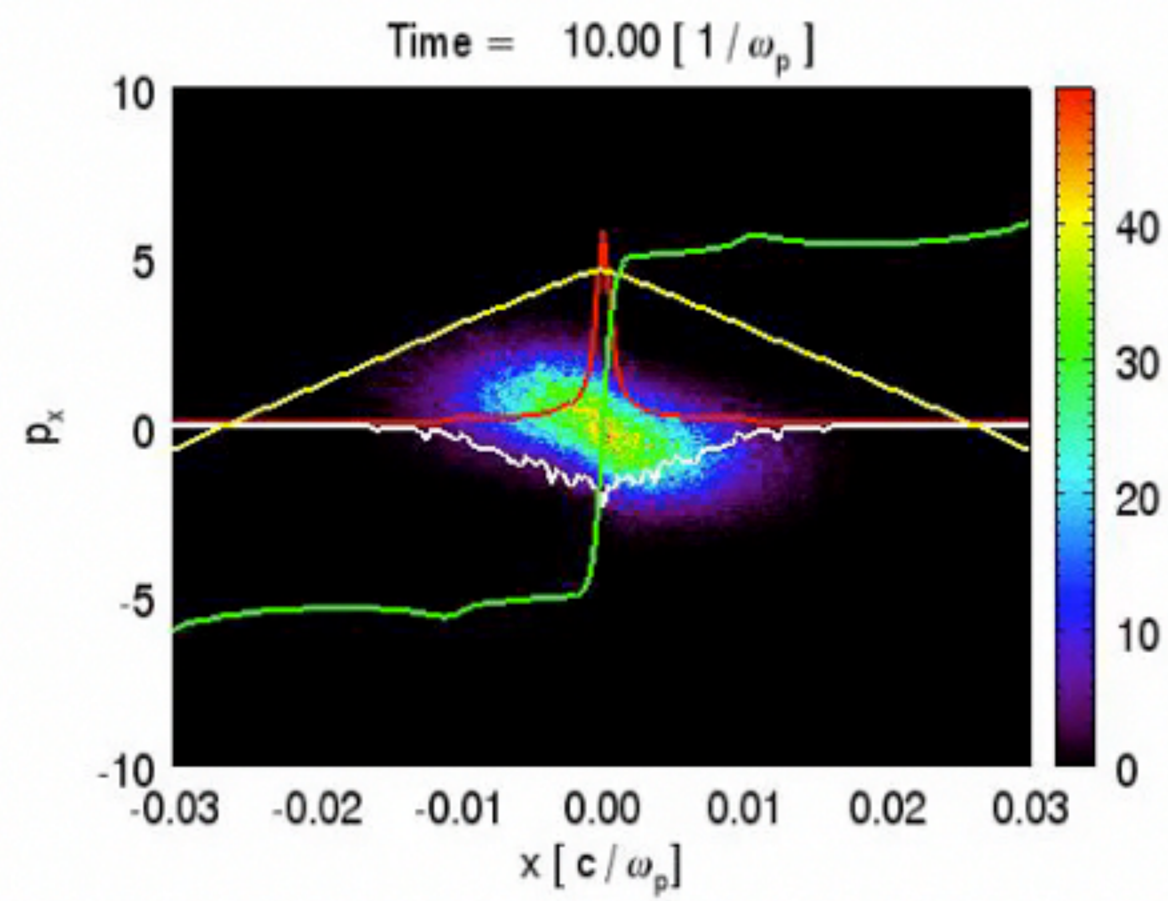
Trailing Beam: σ_r = 0.1 μm (0.006 k_p^{-1}) , σ_z = 10.0 μm , $N_2 = 1.0 \times 10^{10}$, ε = **0.093 mm·mrad**

Distance between two beams : 115 μm Plasma Density : 1.0×10^{17} cm^{-3} (H Plasma)





Slice 5

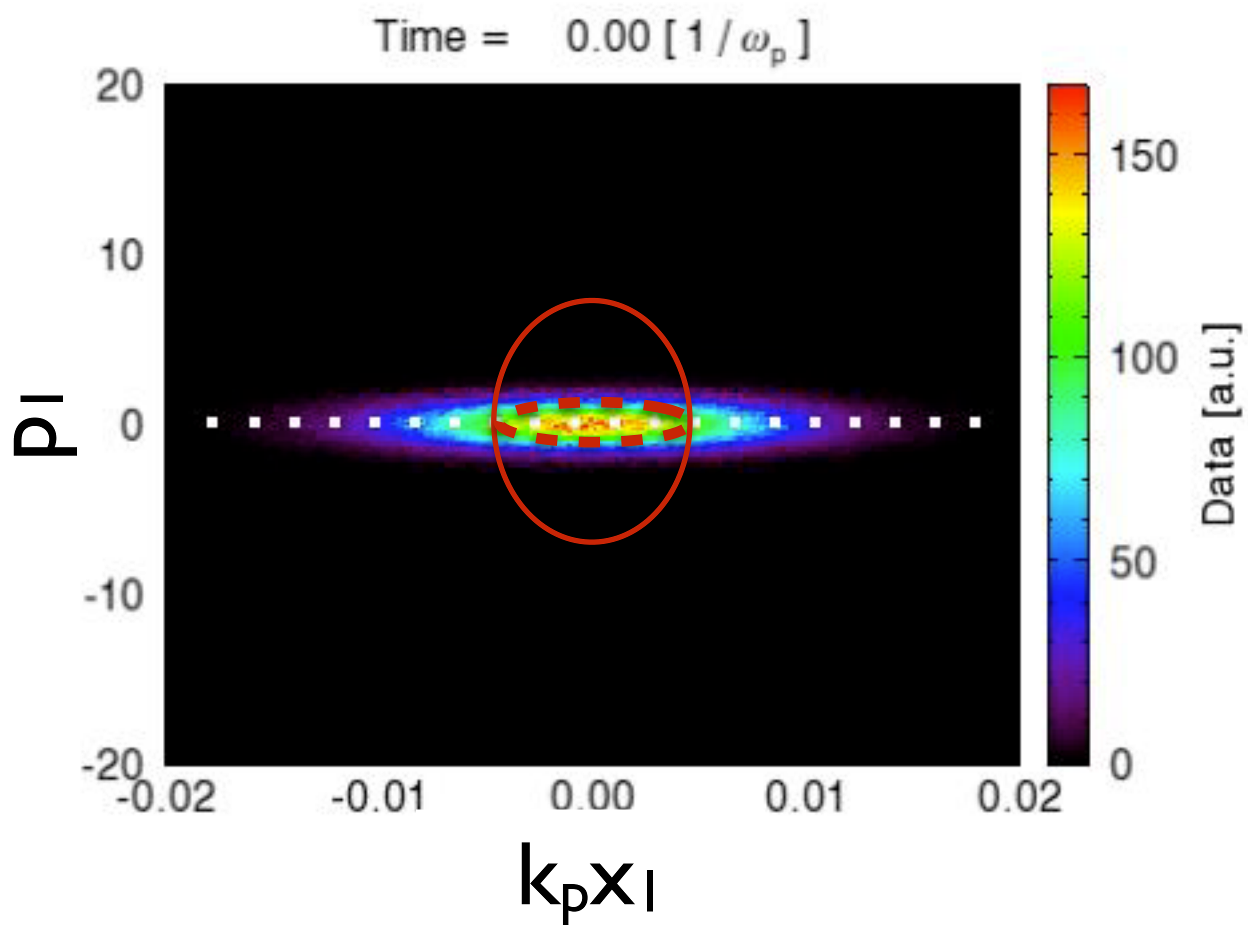


$p_{max}/p_0 \sim 5 \gg \varepsilon_{Nf}/\varepsilon_{N0} \sim 2$

Slice Beam Evolution

Particle's trajectory

Phase Space of p1x1



p_{max}

$p_{time\_averaged}$

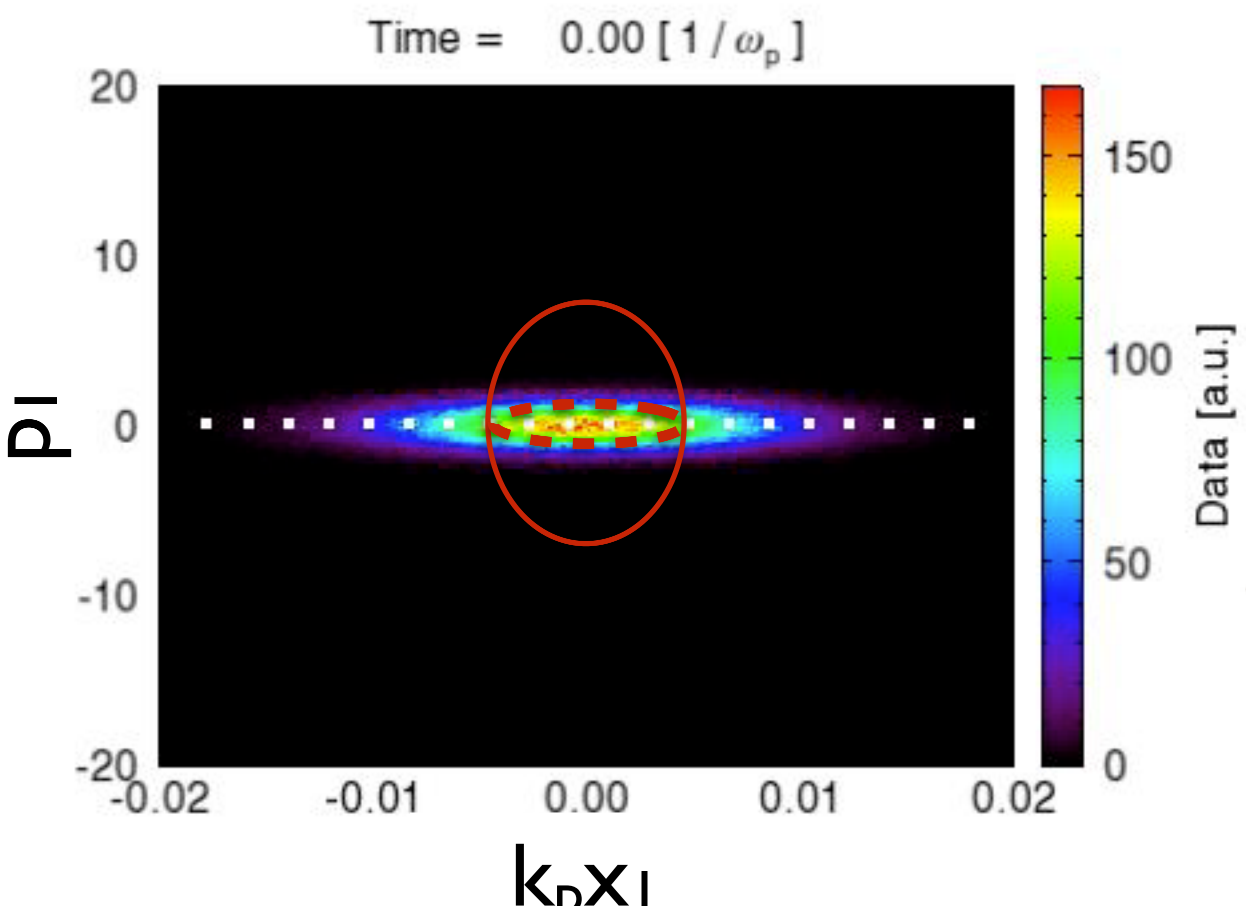
Slice Beam Evolution

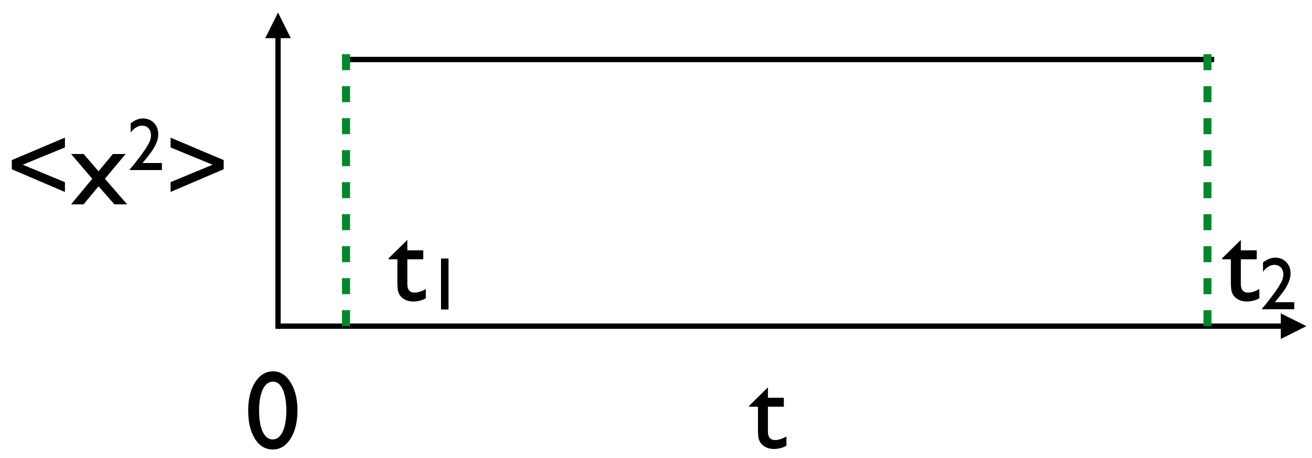
Emittance in the equilibrium state

$$\epsilon_N = \sqrt{< x^2 >< p_x^2 > - < xp_x >^2}$$

$$\epsilon_{Nf} = \sqrt{< x^2 >_f < p_x^2 >_f}$$

Phase Space of p1x1

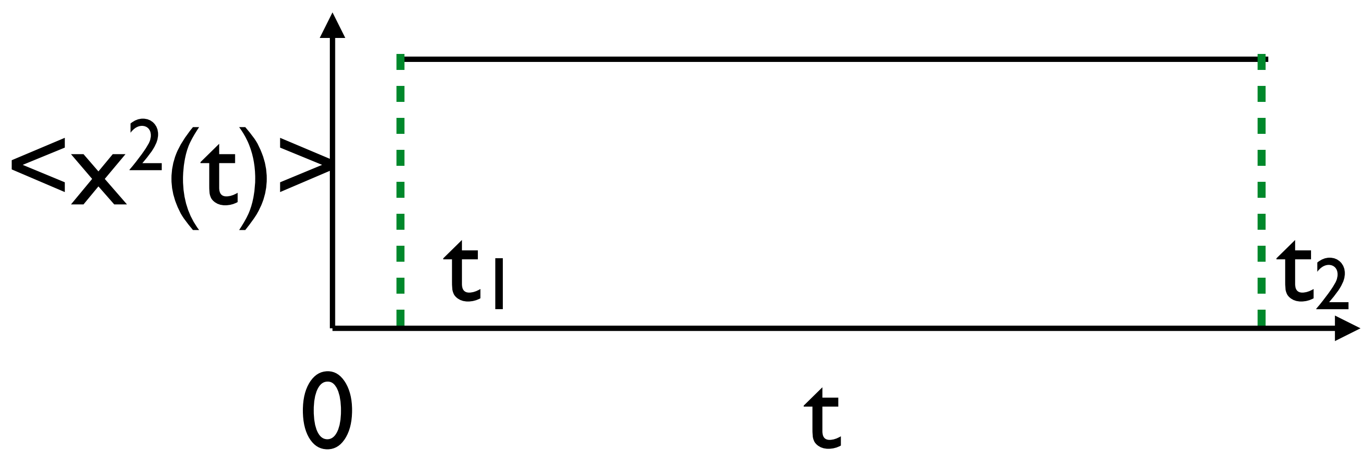




$$< x^2 >_f = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} < x^2 > dt = < \frac{\int_{t_1}^{t_2} x^2 dt}{t_2 - t_1} >$$

Slice Beam Evolution

Emittance in the equilibrium state



$$< x^2 >_f = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} < x^2 > dt = < \frac{\int_{t_1}^{t_2} x^2 dt}{t_2 - t_1} >$$

Each particle

$$\frac{\int_{t_1}^{t_2} x^2 dt}{t_2 - t_1} = \frac{\int_{osc} x^2 dt}{T_{osc}}$$ when $t_2 - t_1 >> T_{osc}$

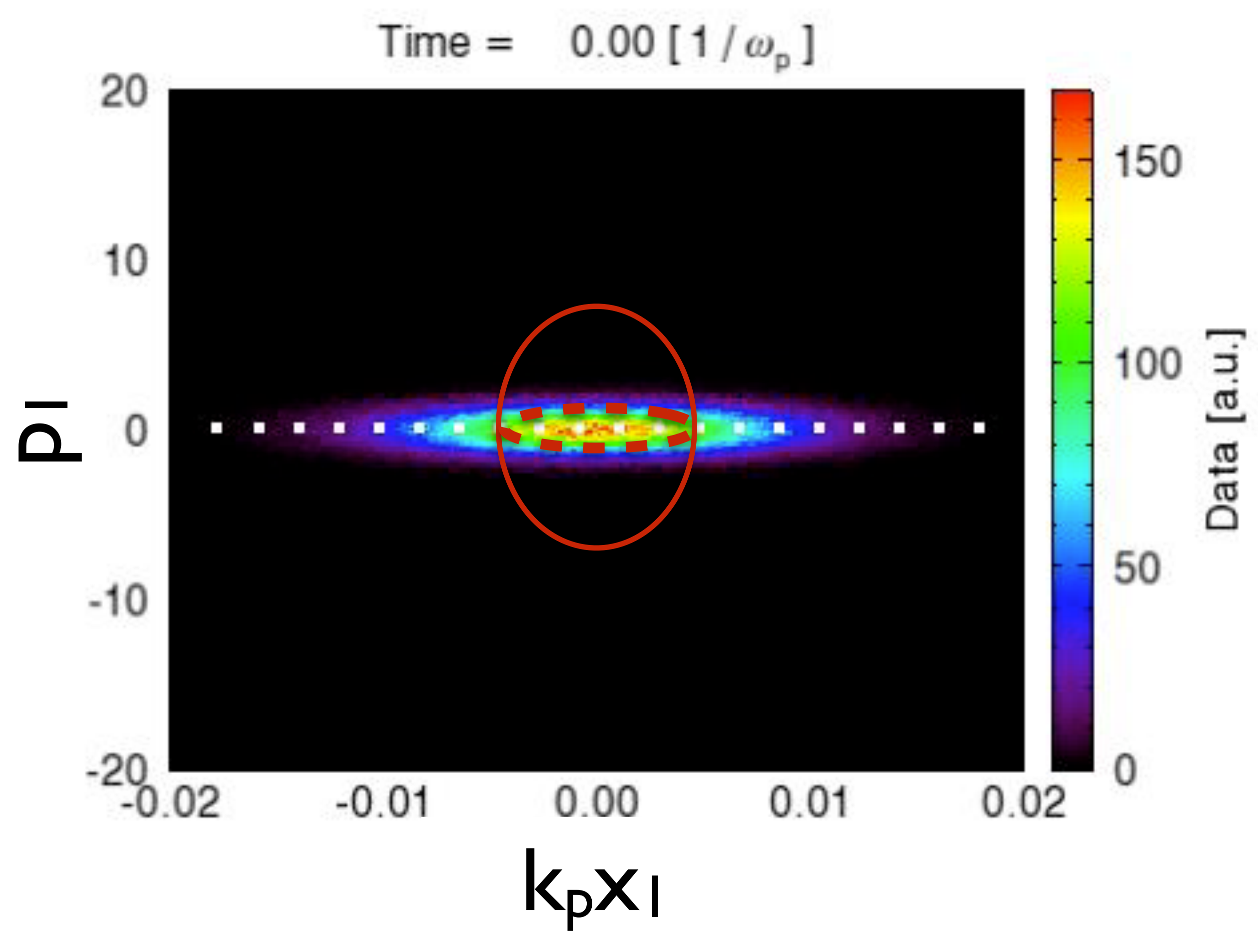
$$< x^2 >_f = < \frac{\int_{osc} x^2 dt}{T_{osc}} >$$

Slice Beam Evolution

$$< x^2 >_f = < \frac{\int_{osc} x^2 dt}{T_{osc}} >$$

Sort the particle according to x_0

$$< x^2 >_f = < \frac{\int_{x_0} x^2 dt}{T_{x_0}} >$$



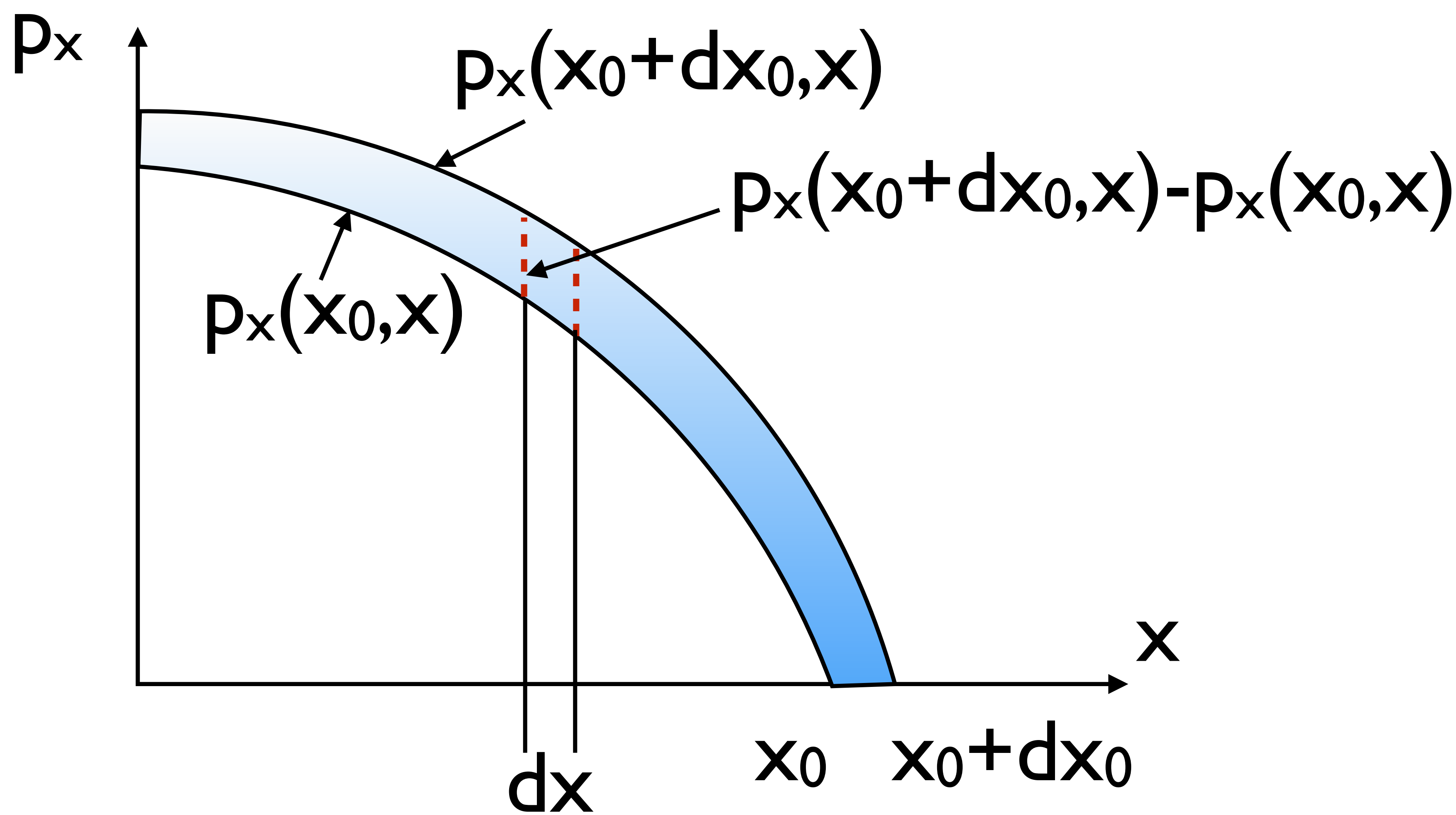
$$< x^2 >_f = < \frac{\int_{x_0} x^2 dt}{T_{x_0}} > = \frac{1}{N} \int_0^\infty N_{x_0} \frac{\int_{x_0} x^2 dt}{T_{x_0}} dx_0$$

N_{x0} does not change because phase space trajectories do not cross with each other.

Slice Beam Evolution

Calculating N_{x0}

$$N_{x0} = 4\int_0^{x_0} f_0(x, p_x)\frac{dp_x}{dx_0}dx$$



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Slice Beam Evolution

$$<x^2>_f = < X_{\mathrm{ave}}^2 > = \frac{1}{N} \int_0^\infty dx_0 N_{x_0} X_{\mathrm{ave}}^2(x_0)$$

$$<p_x^2>_f = \frac{1}{N} \int_0^\infty dx_0 N_{x_0} P_{\mathrm{ave}}^2(x_0)$$

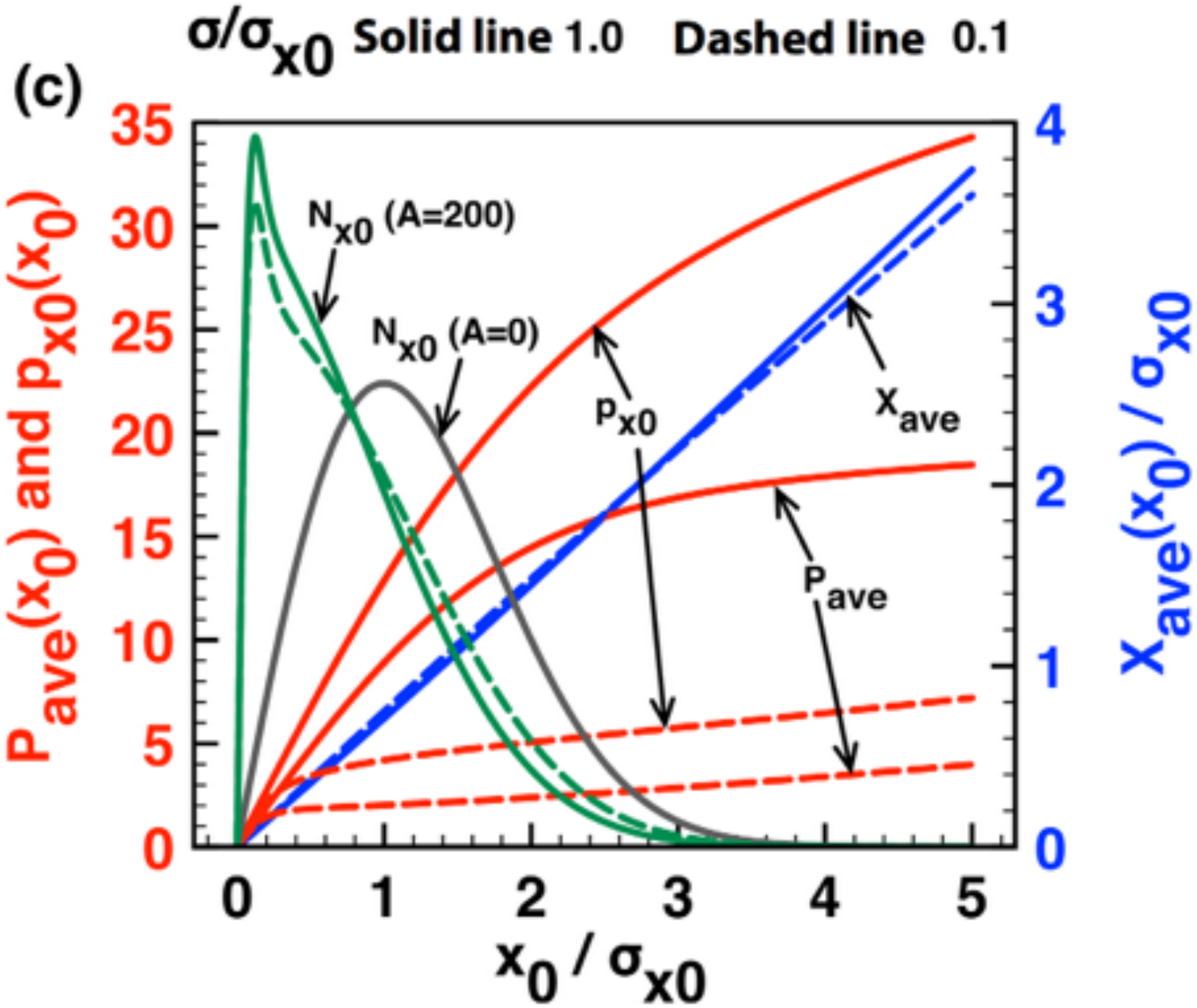
$$N_{x0} = 4 \int_0^{x_0} f_0(x, p_x) \frac{dp_x}{dx_0} dx$$

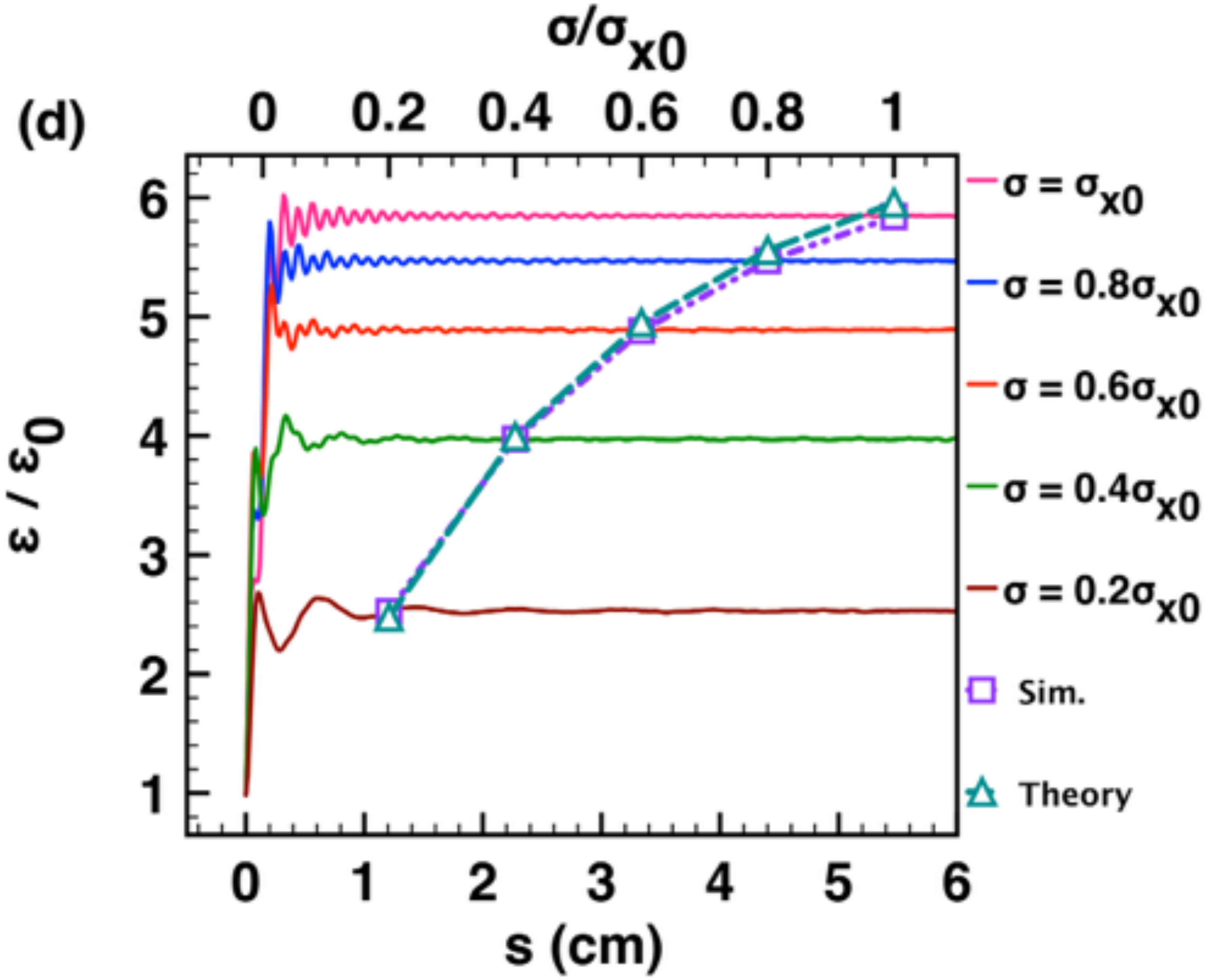
$$X_{\mathrm{ave}}^2 = \frac{\int_0^{x_0} \frac{dx}{v_x} x^2}{\int_0^{x_0} \frac{dx}{v_x}} = \frac{\int_0^{x_0} dx\, x^2 / \sqrt{(\psi(x,\xi) - \psi(x_0,\xi))}}{\int_0^{x_0} dx / \sqrt{(\psi(x,\xi) - \psi(x_0,\xi))}}$$

$$P_{\mathrm{ave}}^2 = \gamma \frac{\int_0^{x_0} dx \sqrt{(\psi(x,\xi) - \psi(x_0,\xi))}}{\int_0^{x_0} dx / \sqrt{(\psi(x,\xi) - \psi(x_0,\xi))}}$$

$$\partial p_x / \partial x_0 = \sqrt{\gamma} F_f(x_0) / \sqrt{2(\psi(x) - \psi(x_0))}$$

Slice Beam Evolution

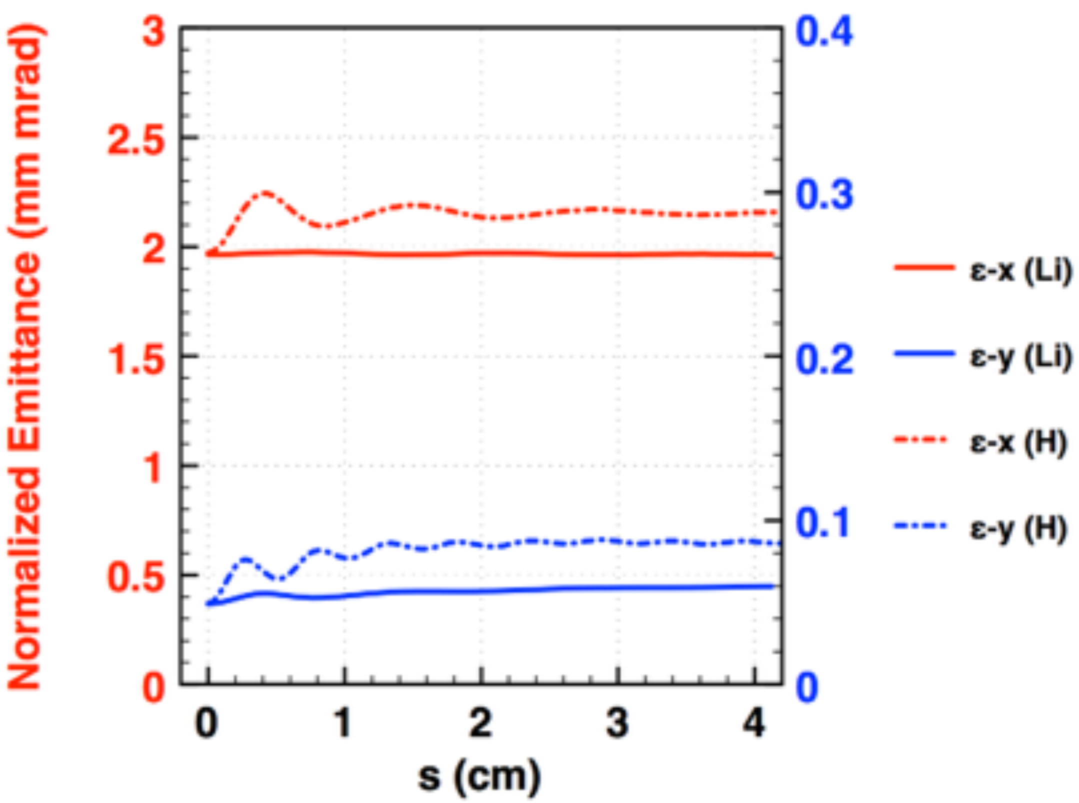


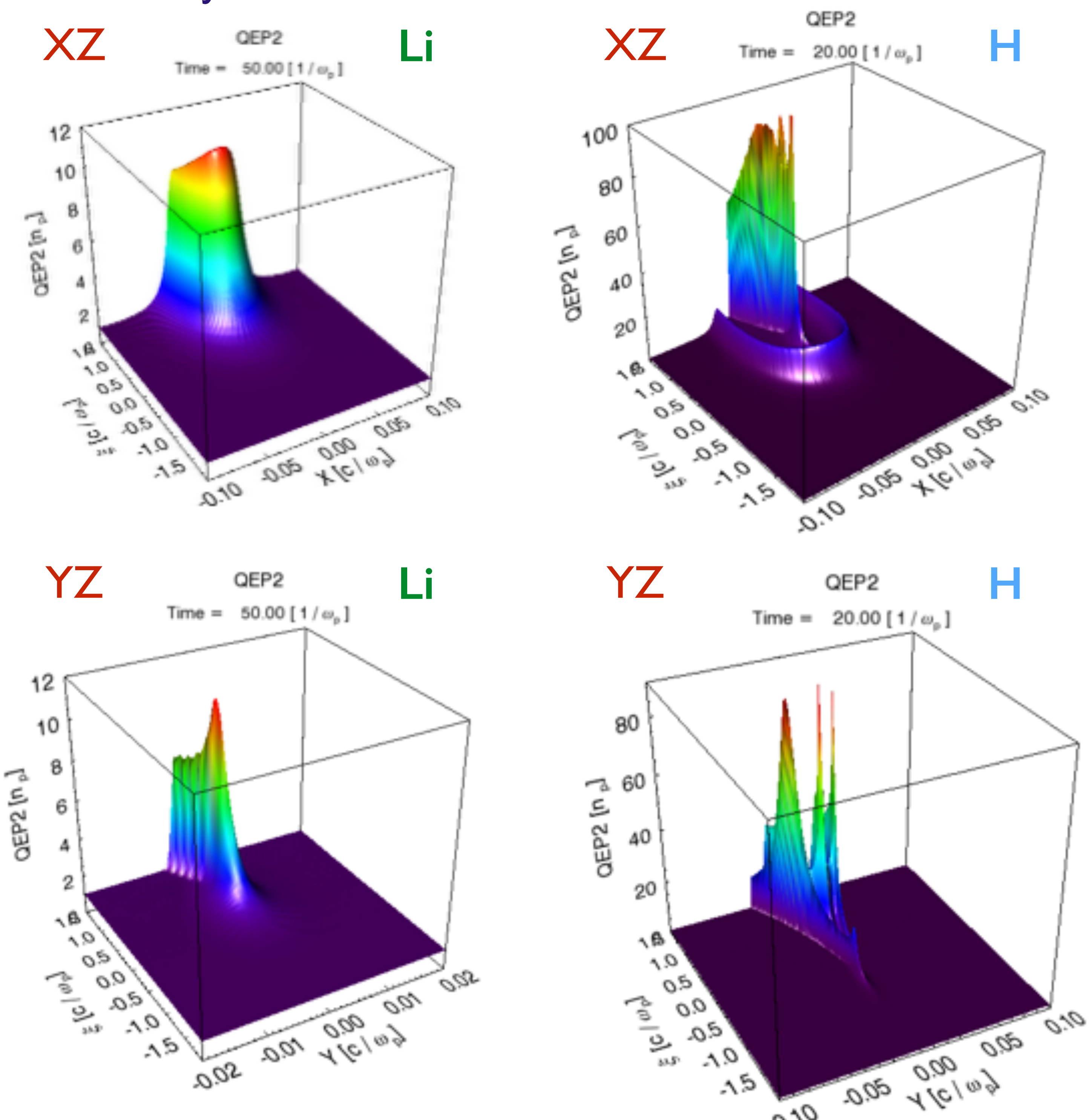


Ion Motion Driven by Asymmetric Trailing Beam

Trailing Beam: σ_z = 10.0 μm , N = 1.0 x 10^{10} ,
σ_x = 0.463 μm , ε_{Nx} = **2.0 mm·mrad** , σ_y = 0.0733 μm , ε_{Ny} = **0.05 mm·mrad**
Y = 48923.7 (25 GeV), Plasma Density : 1.0 x 10^{17} cm^{-3}

$\sigma_x / \Delta_\perp$ = 75.9
$\sigma_y / \Delta_\perp$ = 12.0





In Li, the emittance in x does not change, and in y direction it only increase by 20%.

In H, the emittance in x increase by 10%, and in y direction it increases by 70%.

Over Focused Beam

The emittance growth can be mitigated.