Emittance preservation in plasma-based accelerators with ion motion

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Overview

- Bunch-induced background ion motion in a PA stage:
  - analytic expression for the perturbed wake
  - bunch emittance growth (projected and slice)
  - analytic expression for the emittance growth

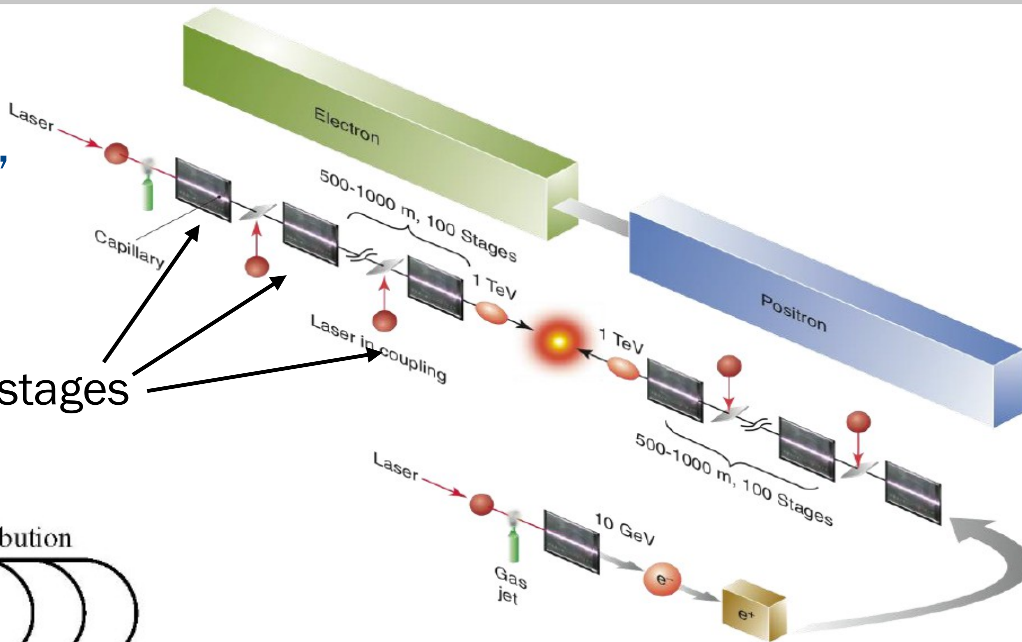
- Ion-motion induced emittance growth suppressed via proper head-to-tail bunch-shaping

- Conclusions

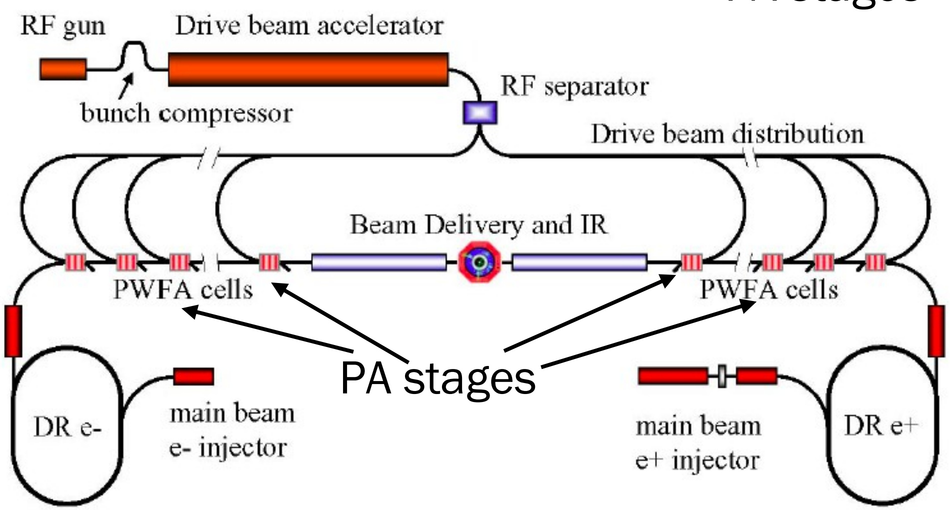
Concept for a TeV-class PA-based linear collider: requires accelerated bunches w/ high charge and small emittance

Laser-driven PA-based LC ———>
[all optical, driver=10s J laser, 10 kHz, 50x10GeV PA stages @ n_0=10^{17} cm^{-3}]

Leemans, Esarey, Physics Today (2009)
Schroeder *et al.*, PRSTAB (2010)



PA stages



PA stages

<—— Beam-driven PA-based LC
[driver=25 GeV e-bunch, 19x25 GeV PA stages @ n_0=10^{17} cm^{-3}]

Seryi *et al.*, PAC 2009
Delahaye *et al.*, IPAC 2014

Luminosity ≥ 10^{34} cm^{-2}s^{-1} → accelerated bunch w/ N_b~ 10^9-10^{10} part., ε_n < 100 nm

*Ellis and Wilson, Nature (2001); Hinchliffe and Battaglia, Phys. Today (2004)

Space-charge field of high-charge, high-energy, linearly matched bunch in a PA can cause background ion motion and perturb transverse wakefield*

- Preservation of bunch emittance during acceleration required

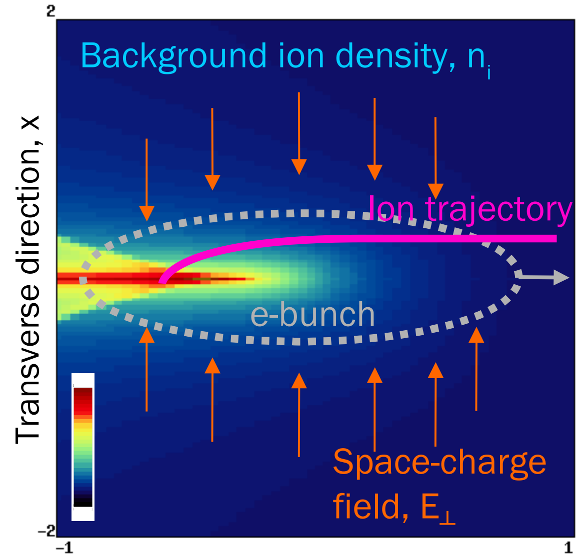
- Emittance preservation achieved via matching bunch size in the linear (unperturbed) confining wake

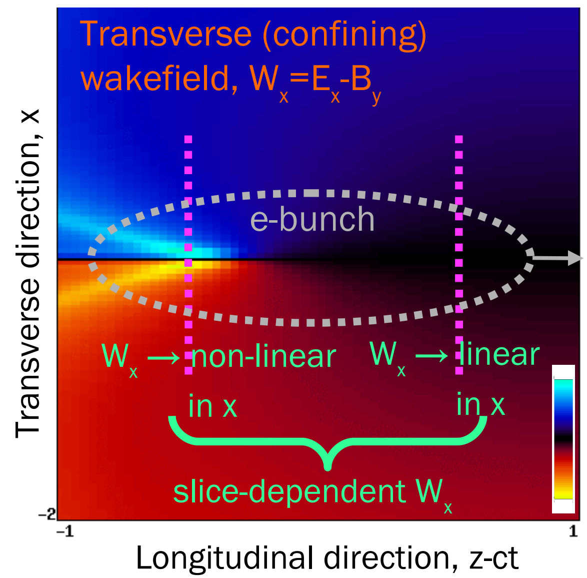
$$\sigma_x^2 = \frac{\epsilon_n}{\gamma k_\beta} = \sqrt{\frac{2}{\gamma}}\frac{\epsilon_n}{k_p}$$

- As γ increases, $\sigma_{x,y}$ adiabatically decrease and n_b increases
  → e-bunch space charge field increases
  → background ions are pulled towards the axis
  → perturbation of the transverse wakefield
  → bunch emittance growth

Condition for ion motion →

$$\Gamma = Z_i(m/M_i)(n_{b,0}/n_0)(k_p L_b)^2 \gtrsim 1$$





*Rosenzweig et al., PRL (2005); An et al., PRL (2017); Benedetti et al., PRAB (2017)

An analytical expression for the perturbed wakefield in presence of ion motion (valid in the non-relativistic regime) has been derived

- Assuming a bunch density of the form

Longitudinal profile

$$n_b(\zeta, \mathbf{r}) = n_{b,0}\, g_{\parallel}(\zeta) g_{\perp}(\mathbf{r}; \zeta)$$

(round beam)

Transverse profile

the perturbed transverse wakefield is

$$\frac{W_r}{E_0} = \frac{k_p r}{2} - Z_i \frac{m}{M_i} \frac{n_{b,0}}{n_0} \frac{k_p^3}{r} \int_{\zeta}^{0} d\zeta' (\zeta - \zeta') g_{\parallel}(\zeta') \int_0^r g_{\perp}(r'; \zeta') r' dr'.$$

(valid as long as ion velocity remains non-relativistic)

- For $g_{\parallel}(\zeta)=1$ for $-L_b \leq \zeta \leq 0$ (and zero elsewhere) and $g_{\perp}(r)=\exp(-r^2/2\sigma_x^2)$

$$\frac{W_r(\zeta, r)}{E_0} = \frac{k_p r}{2} \left[1 + Z_i \frac{m}{M_i} \frac{n_{b,0}}{n_0} \frac{(k_p \zeta)^2}{2} \frac{1 - \exp(-r^2/2\sigma_x^2)}{r^2/2\sigma_x^2} \right]$$

Unperturbed wakefield

Slice-dependent confining force

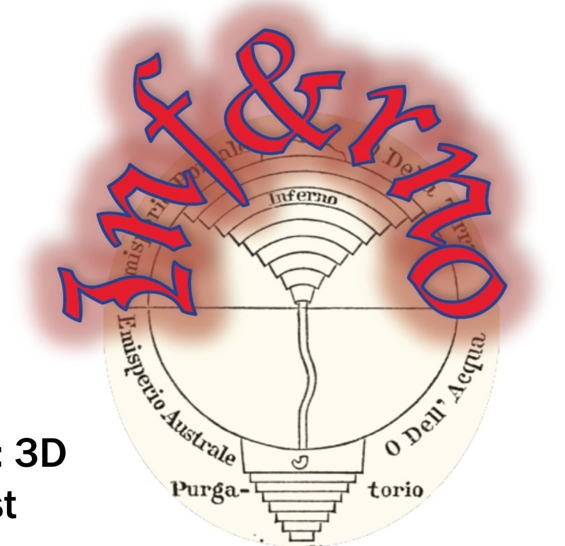
Wakefield acquires nonlinear dependence on transverse coordinate r

Benedetti et al., PRAB (2017)

Modeling performed with INF&RNO*: reduced code tailored to efficiently model PAs → several orders of magnitude faster than full 3D PIC codes still retaining physical fidelity

INF&RNO framework

- Envelope description for laser driver
  - laser → plasma force via averaged ponderomotive force
  - 2D-cylindrical geometry: 3D physics at 2D comp. cost



- PIC or (cold) fluid description for plasma
- "time-explicit" or quasi-static module

Different level of approximation for and complexity of plasma description

- Moving window, parallel, independent grid for laser driver and wakefield, tunnel ionization module, etc.

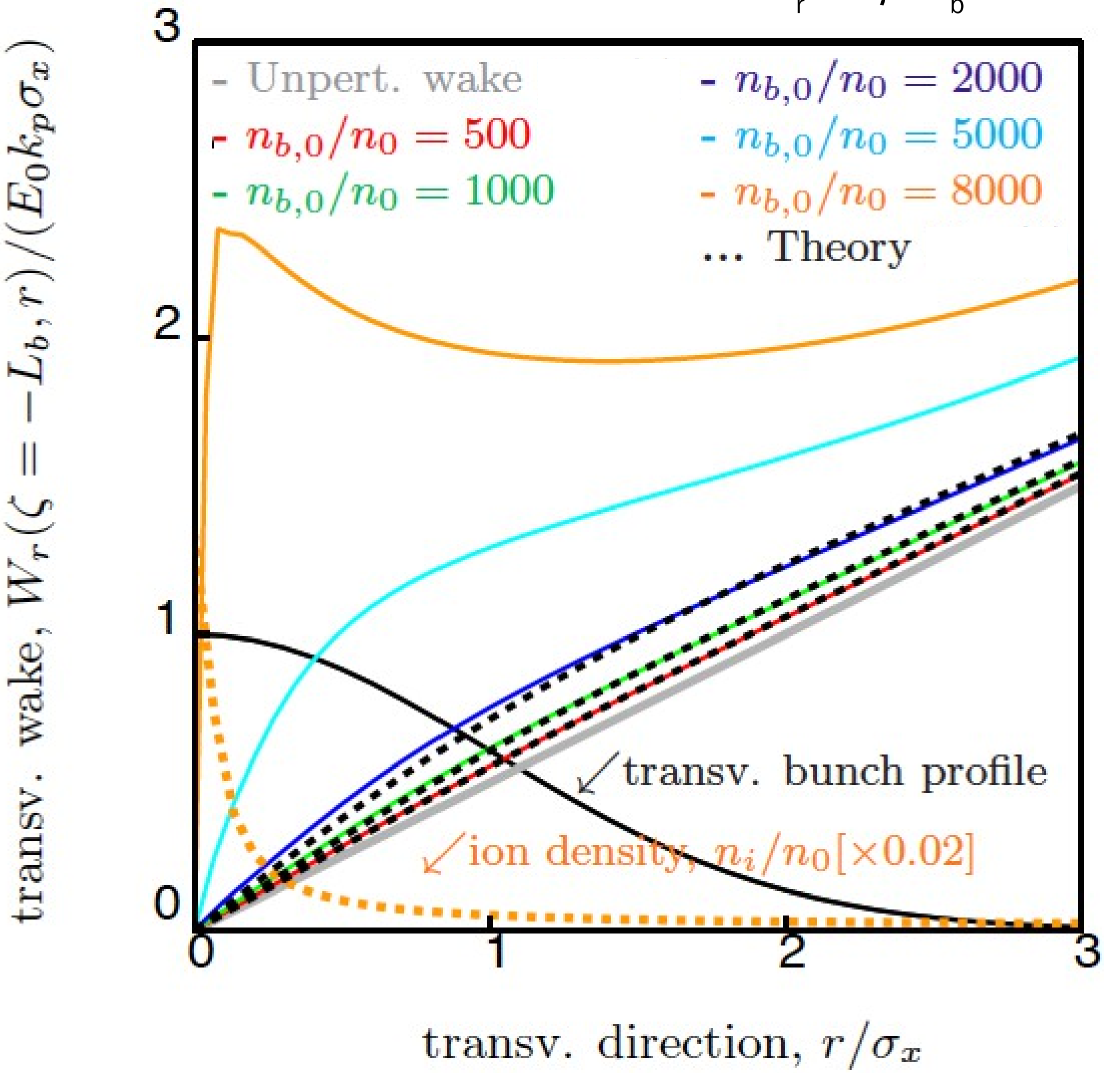
Features to increase the speed-up & additional physics

INF&RNO enables efficient modeling of PAs in a reasonable time (a few hours/days) and on small computers.

*Benedetti *at al.*, AAC2010, AAC2012, ICAP2012, AAC2016, PPCF2017

Analytical result in agreement with modeling in the non-relativistic ion motion regime

Transverse lineout of W_r at ζ=-L_b



(round beam)

Parameters:
n_0=10^{17}cm^{-3} (Hydrogen)
$k_p L_b$=1 (flat-top)
$k_p \sigma_x$=0.015 (Gaussian)

- Analytical solution is in good agreement with modeling for $n_{b,0}/n_0$<2000 (corresponding to Γ<1)
- For $n_{b,0}/n_0$> 2000 (Γ^2>>1) the ion distribution collapses towards the center of the bunch, generating a high-density filament with a characteristic size << σ_x

An et al., PRL (2017)

Benedetti et al., PRAB (2017)

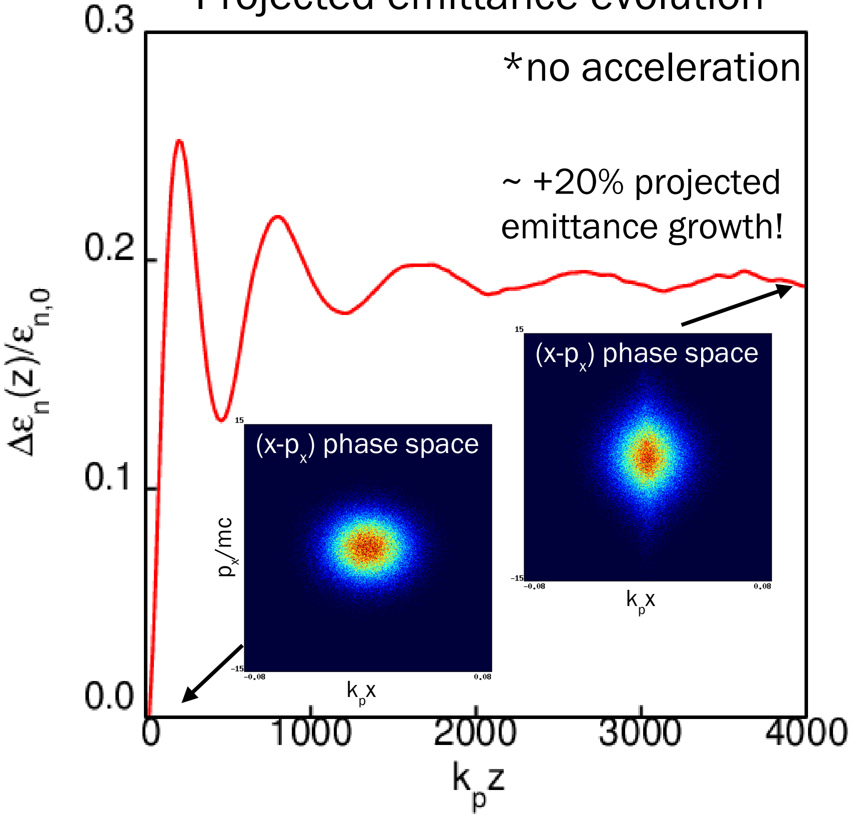
For a bunch initially matched in the linear (unperturbed) wakefield ion motion results in bunch emittance growth

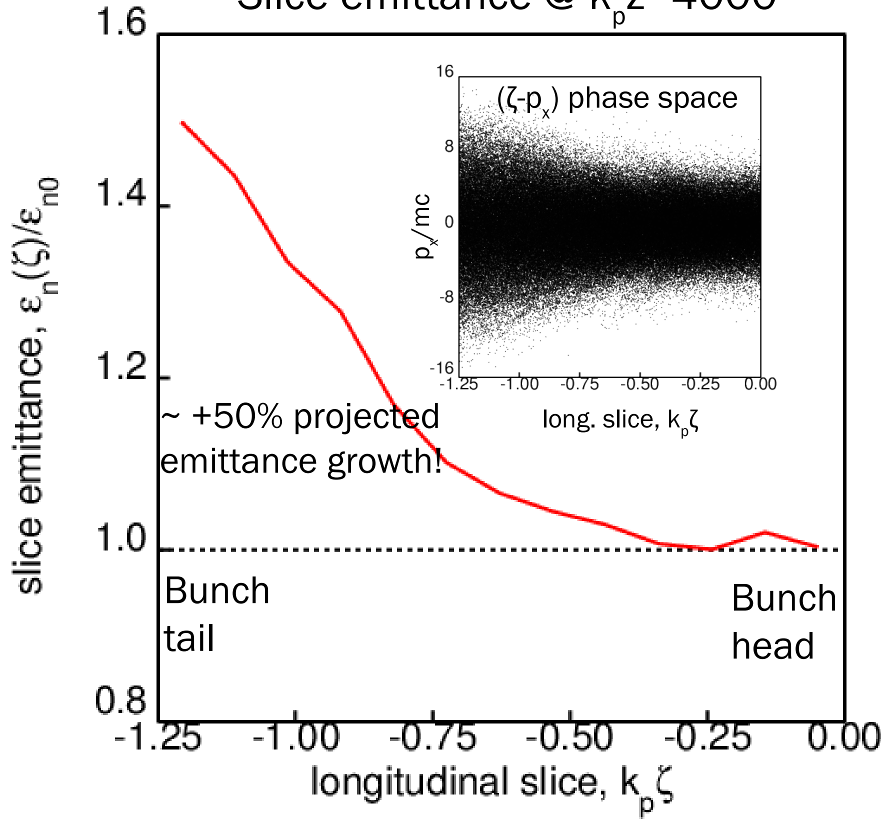
Bunch: E=25 GeV, $\varepsilon_{n,0}=(\varepsilon_{n,x}\varepsilon_{n,y})^{1/2}$=0.6 um, L_b=20 um, $N_b=10^{10}$ ($n_{b,0}/n_0$=12000 → Γ=10)

Background: Hydrogen ions, $n_0=10^{17}$ cm^{-3}

Delahaye *et al.*, IPAC 2014

(round beam)

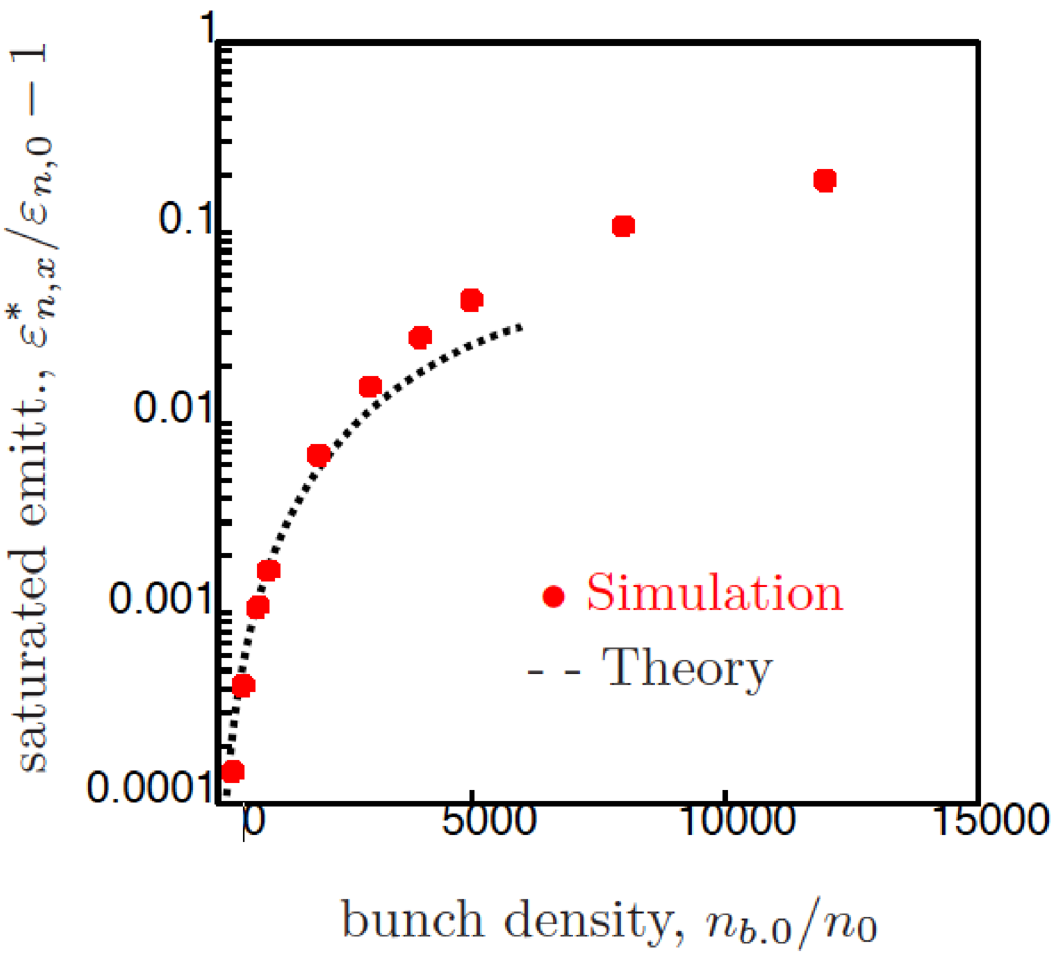




→ tapered bunch emittance not desirable

An analytical expression for the final emittance growth in presence of ion motion (valid in the non-relativistic limit) has been derived

(round beam)



- Bunch:

E=25 GeV

$\varepsilon_{n,0}=(\varepsilon_{n,x}\varepsilon_{n,y})^{1/2}=0.6$ um

L_b=20 um (flat-top)

Charged is varied

(no acceleration)

- Background:

Hydrogen ions

$n_0=10^{17}cm^{-3}$

Benedetti et al., PRAB (2017)

Final (saturated) emittance →
$$\frac{\varepsilon^*_{n,x}}{\varepsilon_{n,0}} = \frac{[\langle x^2\rangle\langle u_x^2\rangle]^{1/2}}{\sigma_x\sigma_{u_x}} \simeq 1 + 0.0015\,\Gamma + 0.001\,\Gamma^2$$

Expressions for the perturbed wakefield and emittance growth at saturation have been derived in the case of flat beams (proposed to reduce beamstrahlung)

- Perturbed wakefield for a flat beam (i.e., $\sigma_x >> \sigma_y$, $\varepsilon_x >> \varepsilon_y$)

(unif. longitudinal profile assumed)

$$\begin{cases} \dfrac{W_x}{E_0} \simeq \dfrac{k_p x}{2} \\ \dfrac{W_y}{E_0} \simeq \dfrac{k_p y}{2}\left[1 + Z_i \dfrac{m}{M_i}\dfrac{n_{b,0}}{n_0}(k_p\zeta)^2 \exp(-x^2/2\sigma_x^2) K(y/\sqrt{2}\sigma_y)\right], \end{cases} \quad K(q) = (\sqrt{\pi}/2)\mathrm{erf}(q)/q$$

Horizontal wake essentially unperturbed

Wake perturbation twice as large compared to the round beam case

- Projected emittance growth at saturation (final)

$$\begin{cases} \dfrac{\epsilon^*_{n,x}}{\epsilon_{n,x}} \simeq 1 \\ \dfrac{\epsilon^*_{n,y}}{\epsilon_{n,y}} \simeq 1 + 0.0027\,\Gamma + 0.0053\,\Gamma^2 \end{cases}$$

Horizontal emittance preserved

Vertical emittance growth twice as large compared to the round beam case

A class of initial beam distributions with constant slice-by-slice emittance enabling ion motion without emittance growth has been derived

The transverse bunch phase-space distribution $f(x, p_x, y, p_y; \zeta; z)$ is chosen to be, slice-by-slice, a stationary solution ($\partial_z f=0$) of the Vlasov equation (VE) including ion motion effects

$$\frac{\partial f}{\partial z} + \{f, H\} = 0 \qquad H = \frac{p_x^2 + p_y^2}{2\gamma_b} + V(x, y; \zeta)$$

Includes perturbation of the wake due to ion motion effects

Hamiltonian describing the transverse dynamics of particles in the bunch

Arbitrary (positively-defined) function

Bunch distribution that is a stationary solution of VE →

$$f(x, p_x, y, p_y; \zeta) = F[H(x, p_x, y, p_y; \zeta)/H_0(\zeta)]$$

Slice-dependent scale parameter

Slice-dependent rms bunch moments →

$$\begin{cases} \overline{x}^2(\zeta) = \mathcal{N}(\zeta)^{-1} \int x^2 f(\mathbf{r}, \mathbf{p}; \zeta) d^2\mathbf{r} d^2\mathbf{p} \\ \overline{p}_x^2(\zeta) = \mathcal{N}(\zeta)^{-1} \int p_x^2 f(\mathbf{r}, \mathbf{p}; \zeta) d^2\mathbf{r} d^2\mathbf{p} \\ \mathcal{N}(\zeta) = \int f(\mathbf{r}, \mathbf{u}; \zeta) d^2\mathbf{r} d^2\mathbf{u} \end{cases}$$

$$\overline{x}^2(\zeta)\overline{p}_x^2(\zeta) = \epsilon_n^2$$

By adjusting the scale factor $H_0(\zeta)$ a constant slice-by-slice emittance is imposed

→ finding the equilibrium solution requires solving self-consistently nonlinear Maxwell-Vlasov system + imposing a constraint on the rms quantities of the bunch distribution

An analytical expression for the equilibrium bunch distribution can be obtained in the non-relativistic regime for particular choices of the function F

Phase space distribution → $f(x,p_x,y,p_y;\zeta) = C\delta[H(x,p_x,y,p_y;\zeta)/H_0(\zeta) - 1]$

Bunch density → $n_b(x,y;\zeta) = n_{b,0}\frac{R_0^2}{R(\zeta)^2}\theta[R(\zeta) - (x^2+y^2)^{1/2}], \quad -L_b \le \zeta \le 0$

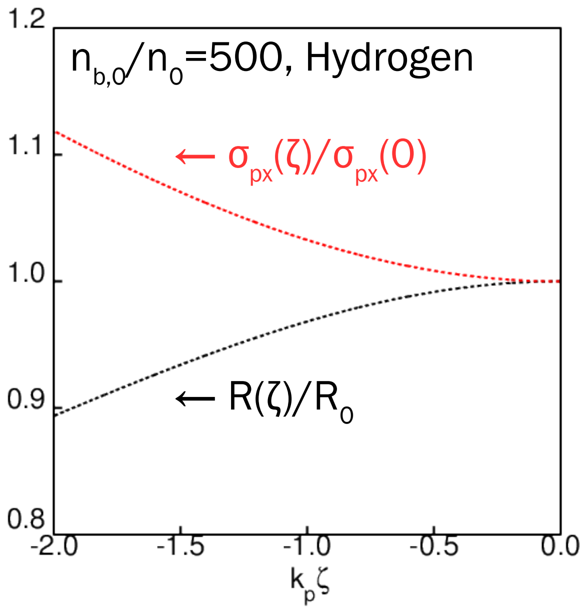
Single particle Hamiltonian → $H = \frac{p_x^2+p_y^2}{2\gamma_b} + \Lambda(\zeta)^2\frac{x^2+y^2}{4}; \quad H_0(\zeta) = \Lambda(\zeta)^2\frac{R(\zeta)^2}{4}$

(round beam)

$$\Lambda(\zeta)^2 = 1 - Z_i\frac{m}{M_i}\frac{n_{b,0}}{n_0}\int_\zeta^0 d\zeta'(\zeta-\zeta')\frac{R_0^2}{R(\zeta')^2}$$

Unperturbed wake

Wake perturbation due to ion motion



Matched bunch radius (including ion motion) → $R(\zeta) = R_0/\Lambda(\zeta)^{1/2}$

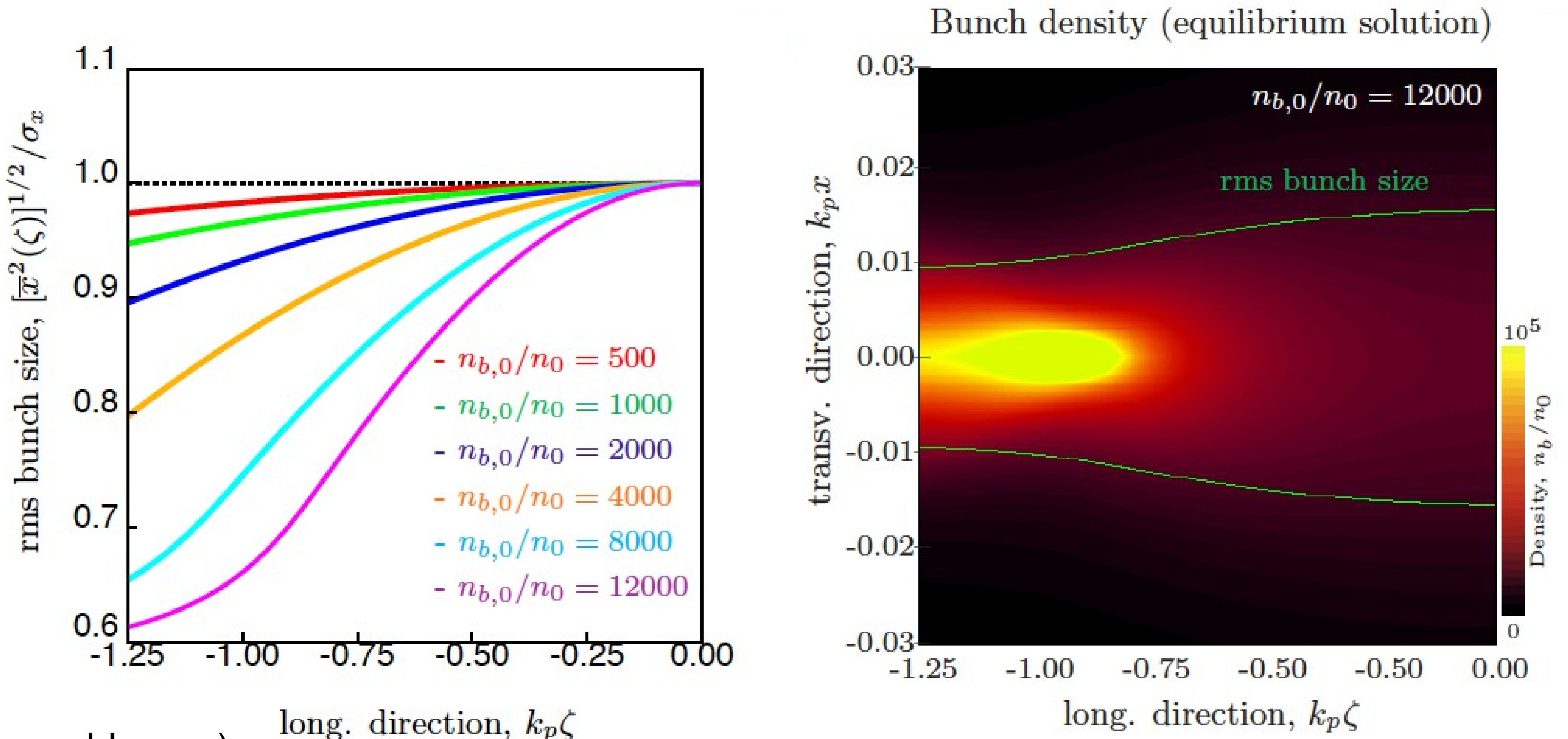
→ Solution in the relativistic ion-motion regime requires numerical solution

Solution for the matched bunch distribution in the relativistic (nonlinear) regime can be found numerically/1

Bunch: E=25 GeV, $\varepsilon_{n,0}=(\varepsilon_{n,x}\varepsilon_{n,y})^{1/2}$=0.6 um, L_b=20 um, $N_b=10^{10}$ ($n_{b,0}/n_0$=12000 → Γ=10)

Background: Hydrogen ions, $n_0=10^{17}$ cm^{-3}

Phase space distribution → $f(x,p_x,y,p_y;\zeta) \sim \exp[-H(x,p_x,y,p_y;\zeta)/H_0(\zeta)]$



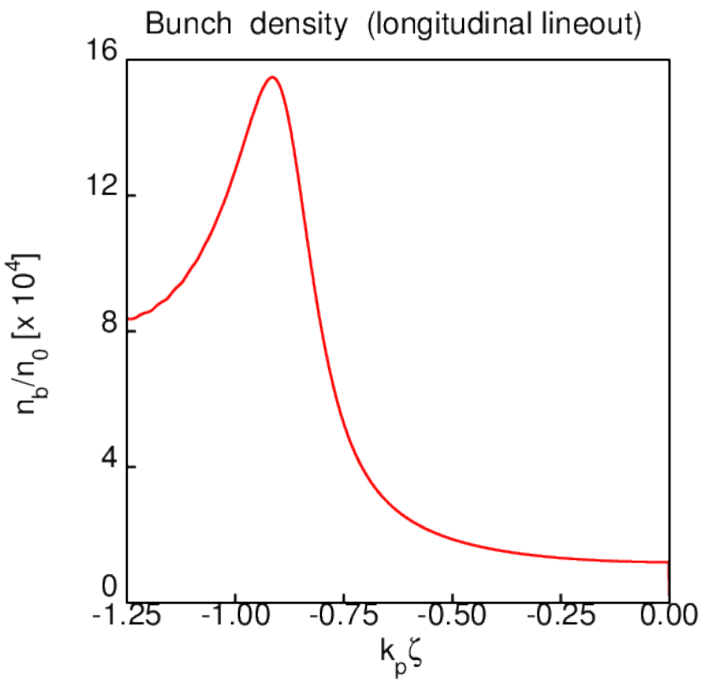
(round beam)

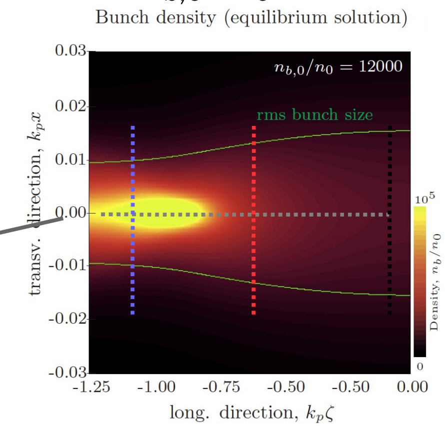
Solution for the matched bunch distribution in the relativistic (nonlinear) regime can be found numerically/2

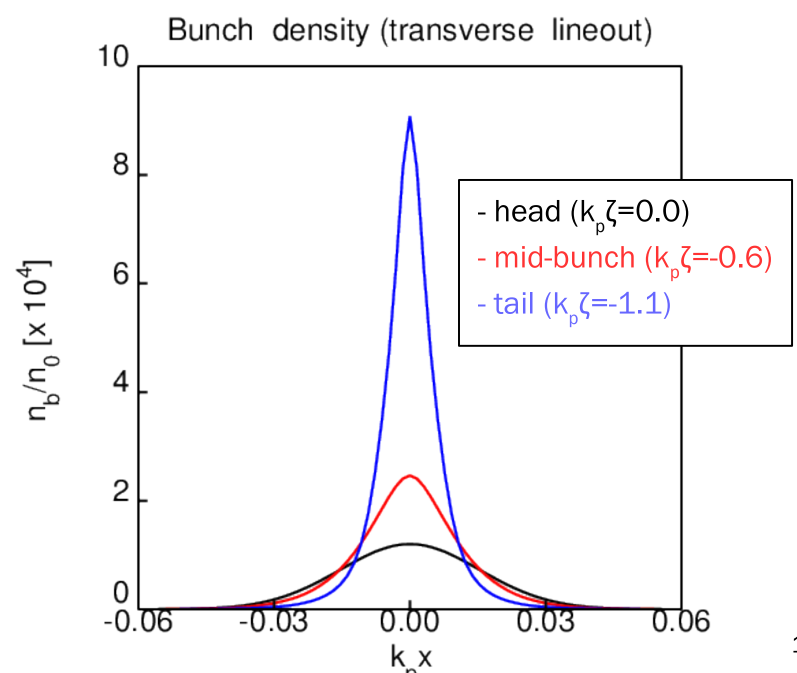
Bunch: E=25 GeV, $\varepsilon_{n,0}=(\varepsilon_{n,x}\varepsilon_{n,y})^{1/2}$=0.6 um, L_b=20 um, $N_b=10^{10}$ ($n_{b,0}/n_0$=12000 → Γ=10)

Background: Hydrogen ions, $n_0=10^{17}$ cm^{-3}

(round beam)







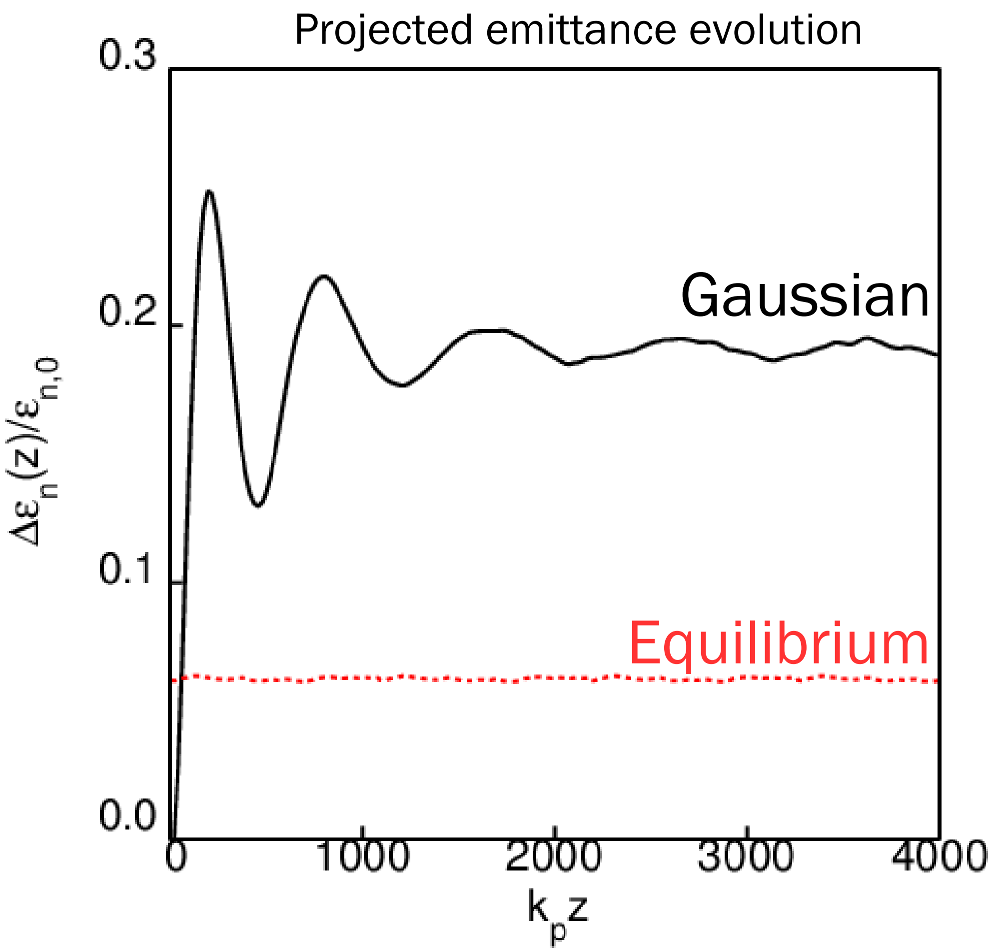
→ Transverse equilibrium bunch distribution is, in general, non-Gaussian

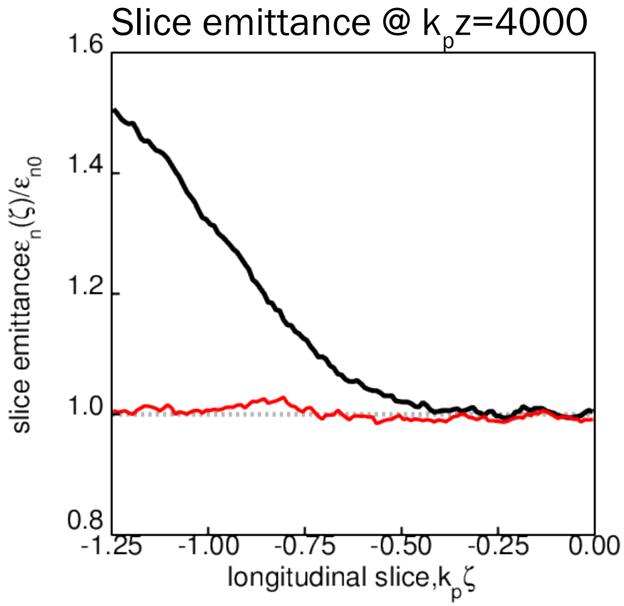
→ Matched solution exists also for a **flat** bunch (i.e., $\sigma_x >> \sigma_y$, $\varepsilon_{n,x} >> \varepsilon_{n,y}$)

Equilibrium solution allows bunch propagation without emittance growth

Bunch: E=25 GeV, $\varepsilon_{n,0}=(\varepsilon_{n,x}\varepsilon_{n,y})^{1/2}$=0.6 um, L_b=20 um, $N_b=10^{10}$ ($n_{b,0}/n_0$=12000 → Γ=10)

Background: Hydrogen ions, $n_0=10^{17}$ cm^{-3}

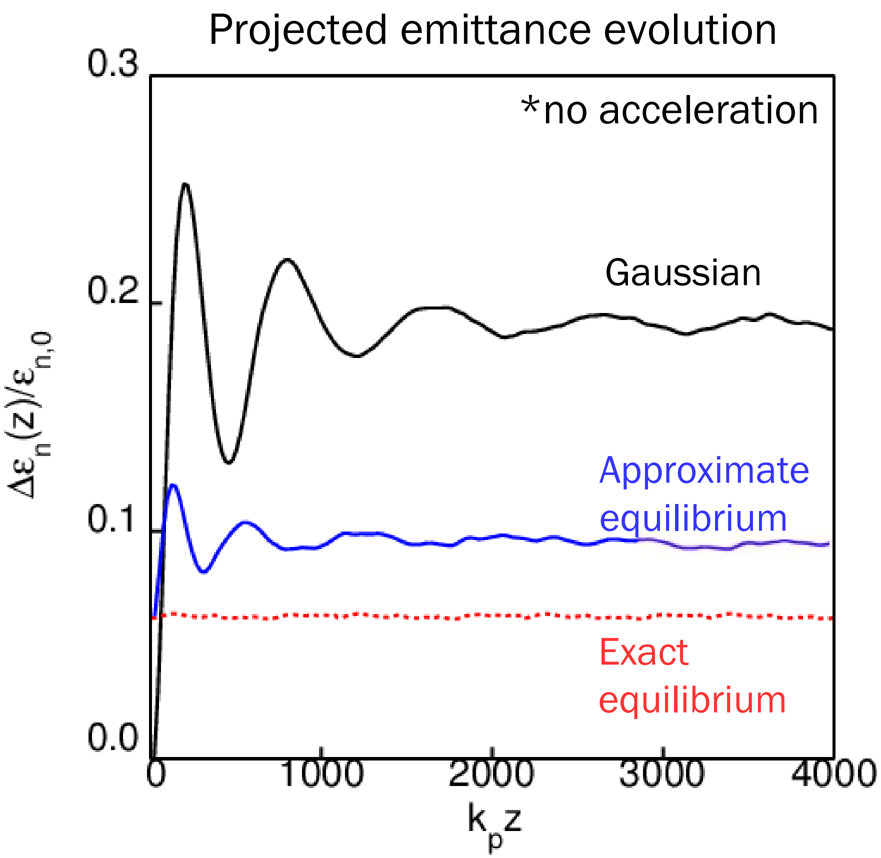


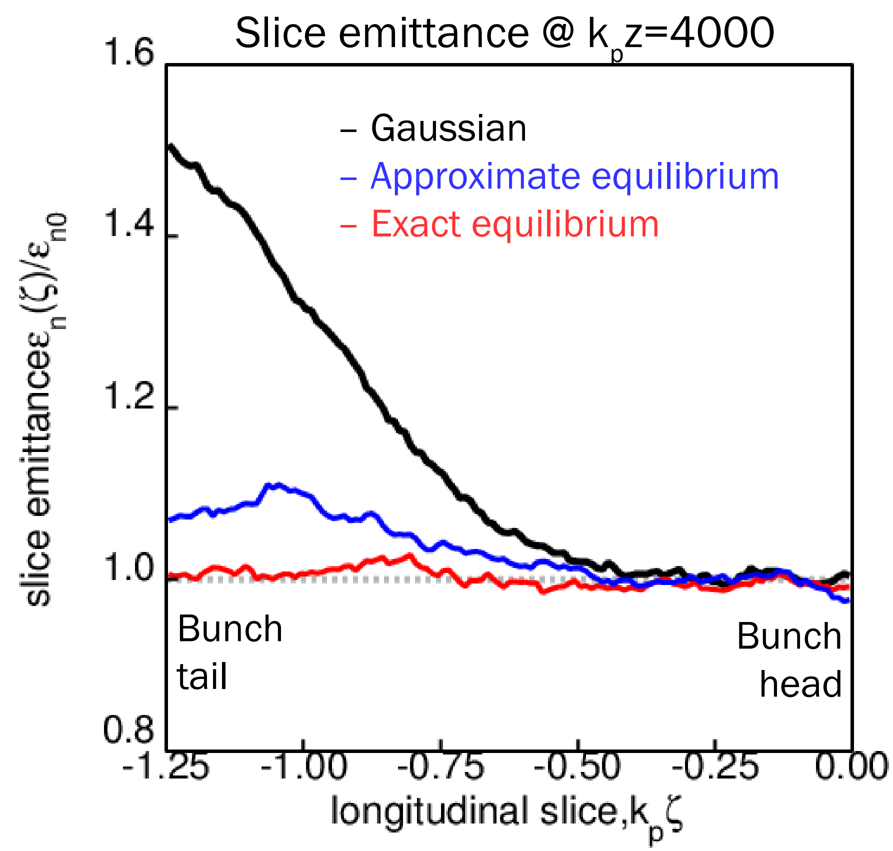


← No emittance growth! [for matched solution slice emittance ≠ projected emittance]

Approximate equilibrium distribution shows moderate emittance growth

Approximate equilibrium → at each longitudinal slice the exact density distribution in the transverse plane is replaced by a Gaussian distribution having same rms properties





→ projected emittance degradation is small (~3%)

→ slice emittance degradation is <10%, much smaller than in the non-tailored case

Conclusions

- The problem of bunch-induced ion motion and the associated bunch emittance growth for a relativistic bunch propagating in an ion column has been studied for parameters relevant to the design of future PA-based LCs

- Analytical expressions for the structure of the transverse wake and for the ion-motion-induced bunch emittance growth have been obtained. Analytical results are in good agreement with numerical modeling performed with INF&RNO.

- A solution that completely eliminates ion-motion-induced emittance growth has been proposed and analyzed. This solution requires a head-to-tail shaping of the bunch distribution.

- Controlling emittance growth is critical to high-energy physics applications of plasma accelerators.