# Emittance Growth Measurement of a 20 GeV beam with FACET I Magnet Config

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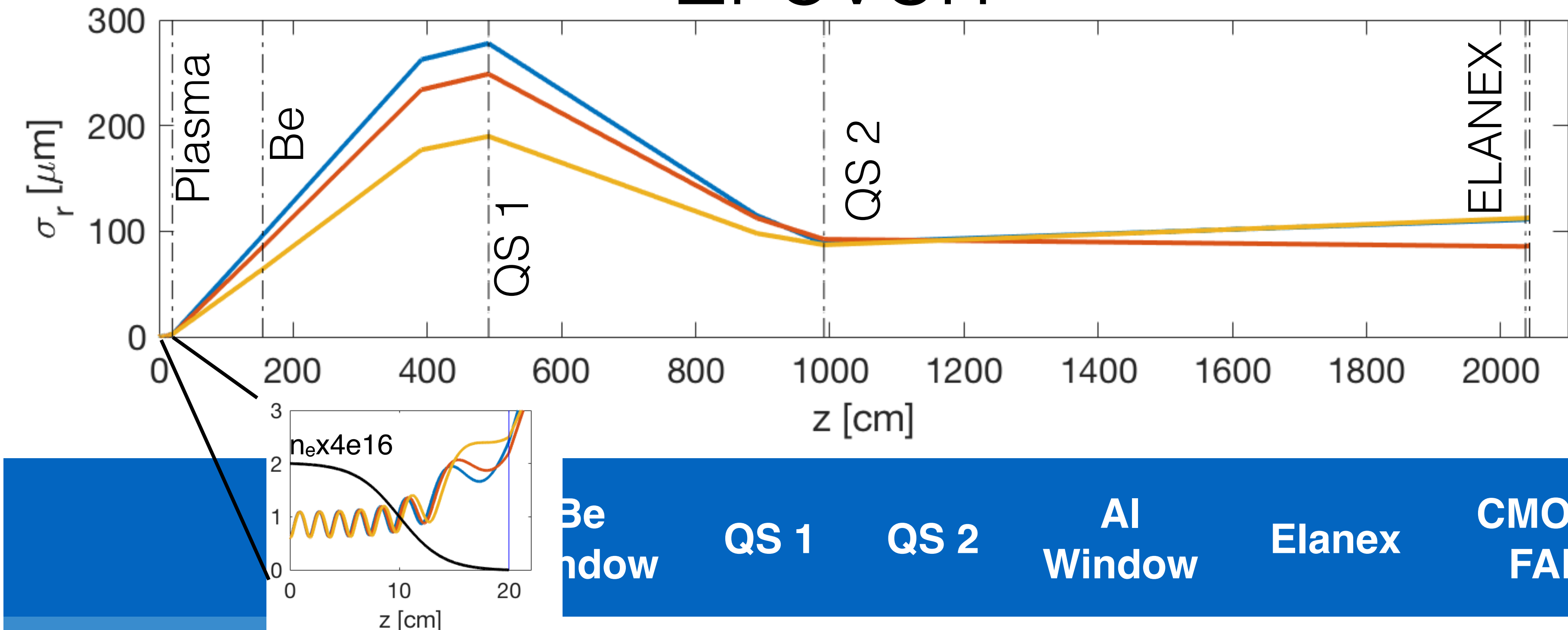
# FACET I Spectrometer Layout with Li oven




| | | Be ndow | QS 1 | QS 2 | Al Window | Elanex | CMOS_FAR |
|---|---|---|---|---|---|---|---|
| **Relative Z (m)** | 0 | 1.55 | 5.91 | 9.91 | 20.31 | 20.37 | 21.19 |
| **Linac Z (m)** | 1994.85 | 1996.4 | 2000.76 | 2004.76 | 2015.16 | 2015.22 | 2016.04 |

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# Models

## Evolution of Twiss Parameters

From Transfer Matrix to evolution of Twiss parameters

$$R = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \longrightarrow \begin{bmatrix} \beta_f \\ \alpha_f \\ \gamma_f \end{bmatrix} = \begin{bmatrix} C^2 & -2CS & S^2 \\ -CC' & CS' + SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{bmatrix} \begin{bmatrix} \beta_i \\ \alpha_i \\ \gamma_i \end{bmatrix}$$

Free Propagation in Space:

$$\begin{bmatrix} x_f \\ x_f' \end{bmatrix} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ x_i' \end{bmatrix} \longrightarrow \begin{bmatrix} \beta_f \\ \alpha_f \\ \gamma_f \end{bmatrix} = \begin{bmatrix} 1 & -2L & L^2 \\ 0 & 1 & -L \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_i \\ \alpha_i \\ \gamma_i \end{bmatrix}$$

$\epsilon_n$, $\sigma_r$, $\alpha$ and distance of Be foil from downramp are optimized to produce the fit

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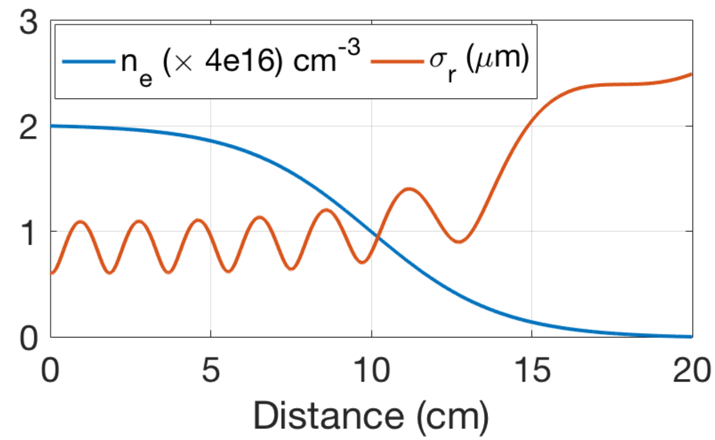
# Models

Plasma Ramp: Continuous focusing element with

$$k = k_{beta} = \frac{\omega_p}{c\sqrt{2\gamma}}$$

$$R = \begin{bmatrix} \cos(k\,dz) & \frac{1}{k}\sin(k\,dz) \\ -k\sin(k\,dz) & \cos(k\,dz) \end{bmatrix}$$




Be/Al foil: Multiple Scattering Angle $\Delta\theta$*

$$\epsilon_f = \sqrt{\epsilon_i(\epsilon_i + \beta_i \Delta\theta^2)}$$

$$\beta_f = \frac{\epsilon_i \beta_i}{\sqrt{\epsilon_i(\epsilon_i + \beta_i \Delta\theta^2)}}$$

$$\alpha_f = \frac{\epsilon_i \alpha_i}{\sqrt{\epsilon_i(\epsilon_i + \beta_i \Delta\theta^2)}} \qquad \gamma = \frac{1+\alpha^2}{\beta}$$

*$\Delta\theta$= 8.3 μrad for 75 μm Be window and 134 mrad for 5 mm Al

Quadrupole Focusing, $k = \frac{1}{\sqrt{Lf}}$

QS1 $$R = \begin{bmatrix} \cos(\sqrt{k}\,L) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}\,L) \\ -\sqrt{k}\sin(\sqrt{k}\,L) & \cos(\sqrt{k}\,L) \end{bmatrix}$$

QS2 $$R = \begin{bmatrix} \cosh(\sqrt{k}\,L) & \frac{1}{\sqrt{k}}\sinh(\sqrt{k}\,L) \\ -\sqrt{k}\sinh(\sqrt{k}\,L) & \cosh(\sqrt{k}\,L) \end{bmatrix}$$

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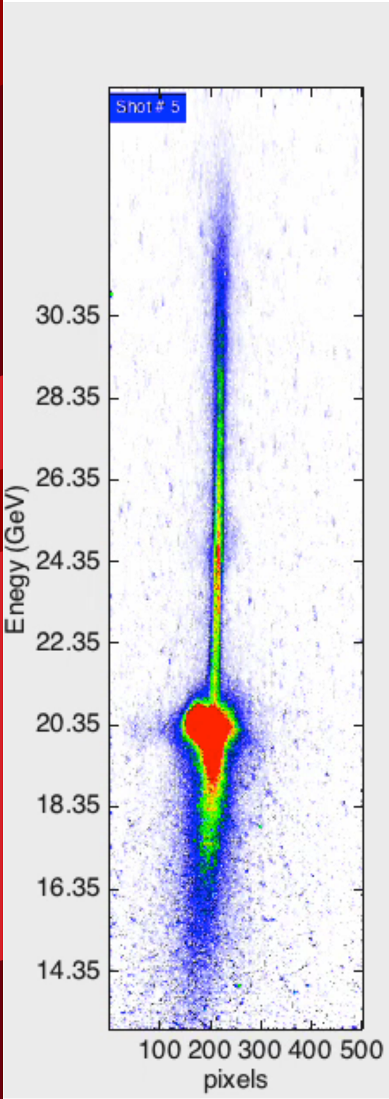
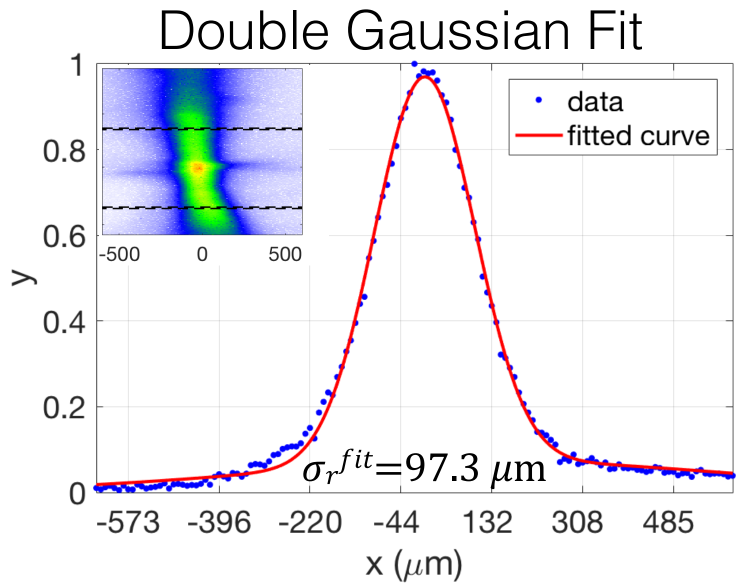
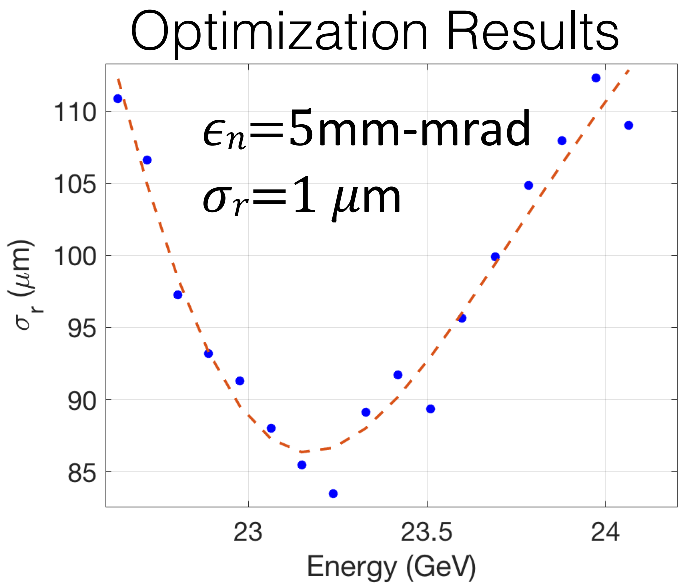
# Injected Beam Emittance at ~23 GeV




- Energy slices equivalent except for energy
- Elements affecting beam size measurement
  - Plasma down-ramp
  - 75 µm Be window
  - Two Quadrupole magnets (QS1,QS2)
  - 5 mm Al window
- $\epsilon_n$, $\sigma_r$, α and optimized to produce the fit

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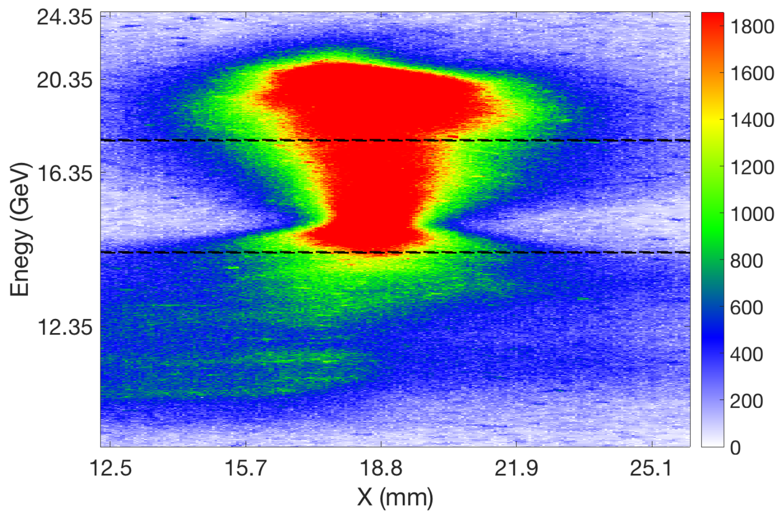
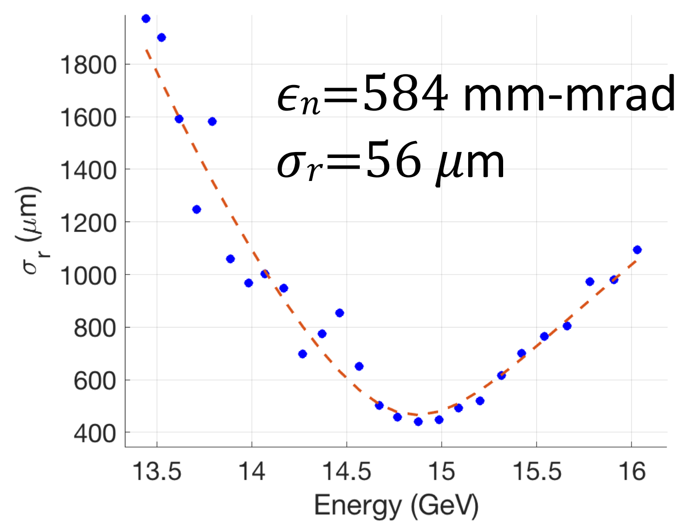
# Drive Beam Emittance at ~15 GeV




- Energy slices equivalent except for energy
- Elements affecting beam size measurement
  - Plasma down-ramp
  - 75 μm Be window
  - Two Quadrupole magnets (QS1,QS2)
  - 5 mm Al window
- $\epsilon_n$, $\sigma_r$, α and optimized to produce the fit

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# Measuring Emittance of a Beam Accelerated to 20 GeV

Configuration

- Magnet/foil locations: Same as FACET I
- Quad strengths: 258.03,-172.06.
  - Calculated for imaging at 20.35 GeV, using one of the DAQ functions.*
- Energy for trailing electron beam: 20 GeV ± 1 GeV
- Butterfly Profile is plotted for different normalized emittance values and for different initial beam sizes

*Image plane at ELANEX (z=2015.22), object plane at 12 cm upstream of the exit of the 1.5 m lithium oven (z=1997.85) — same config as last 1.5 m positron run

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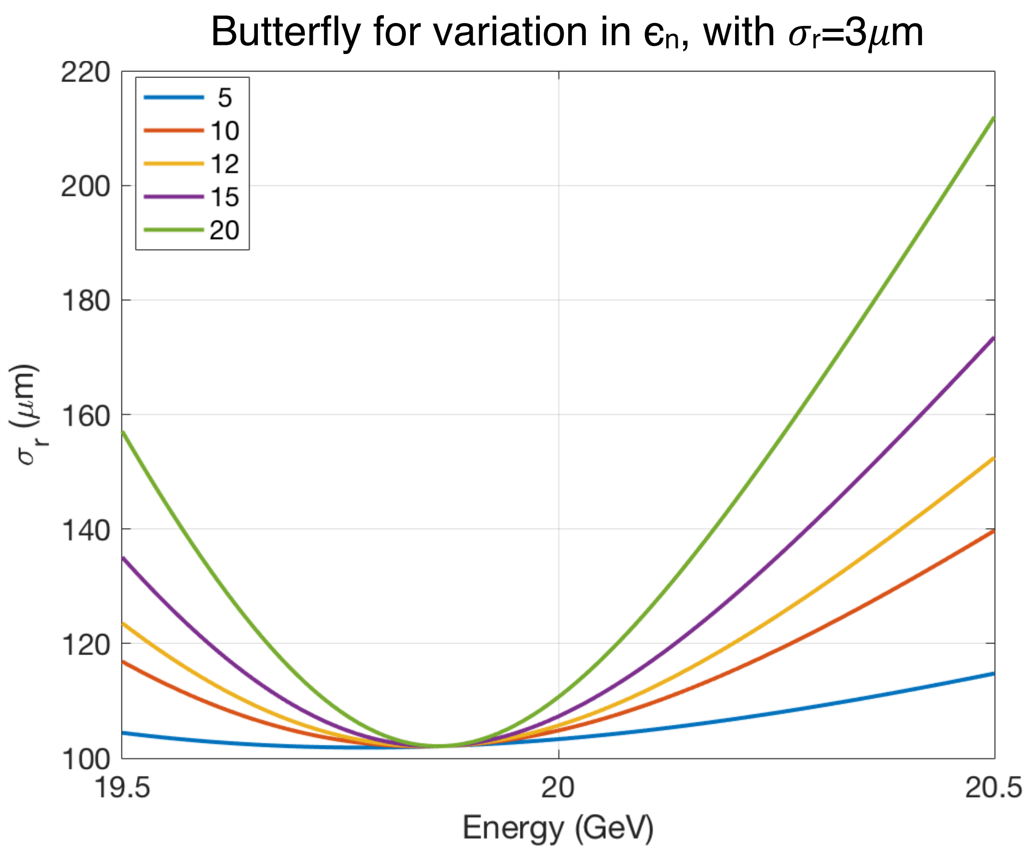
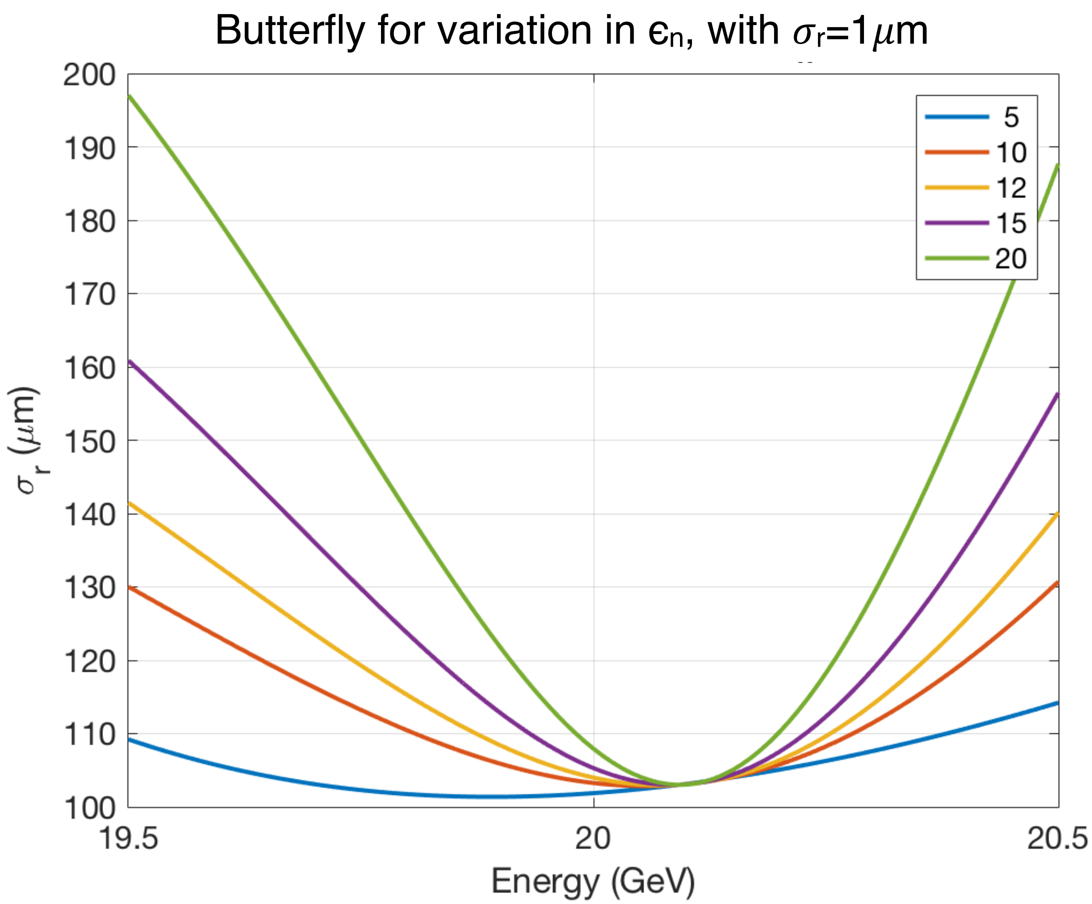
# Emittance of 20 GeV Beam From the End of the Matching Section




In a 1 GeV window, the different cases within a factor of two can be distinguished relatively easily

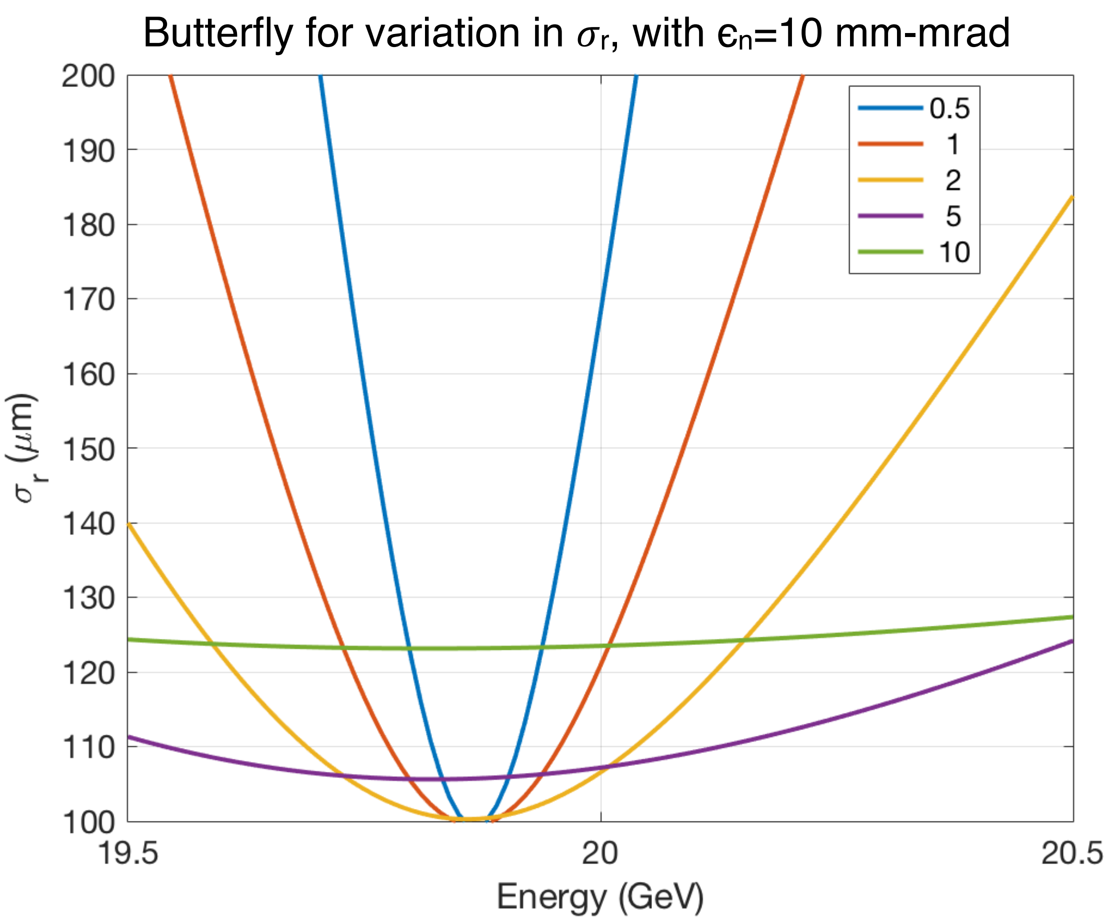
# Emittance of 20 GeV Beam With FACET I Lithium Ramp




Peak density is assumed to be $5 \times 10^{16}$ $cm^{-3}$

Ramp profile is the same as long plasma Li profile

# Highly Unmatched Beams in Plasma




However, the larger the initial beam size is, the distinction becomes more difficult to make

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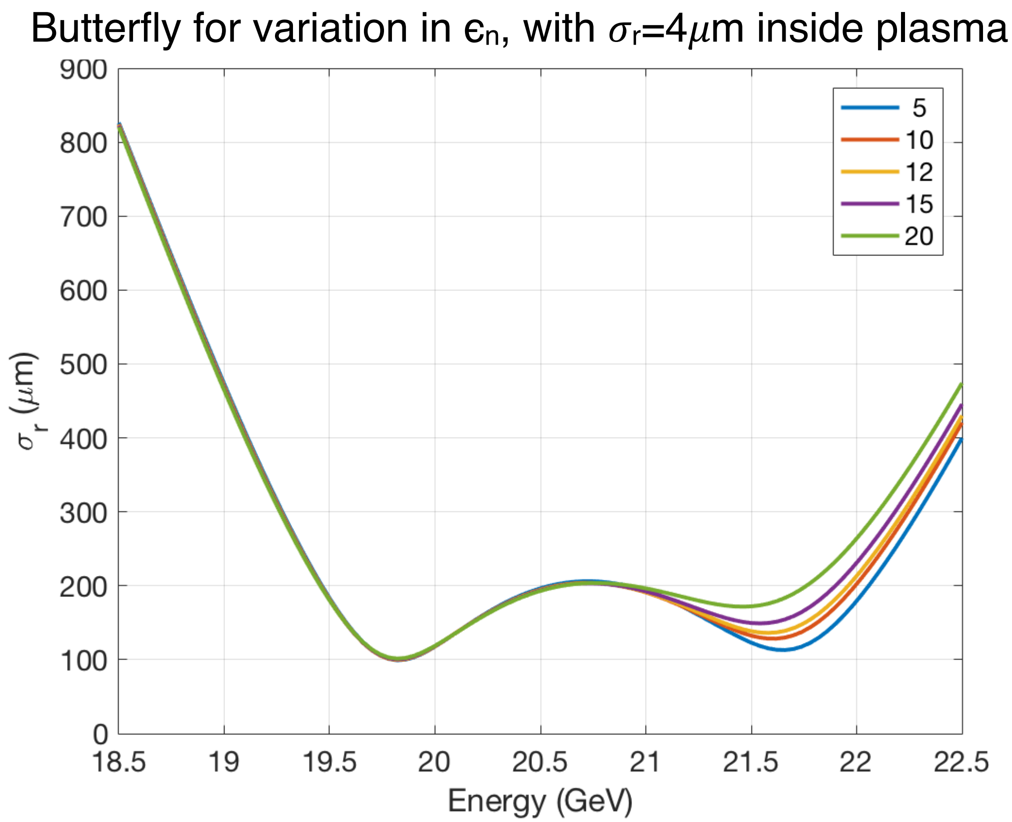
# Butterfly Feature for 4µm Initial Size



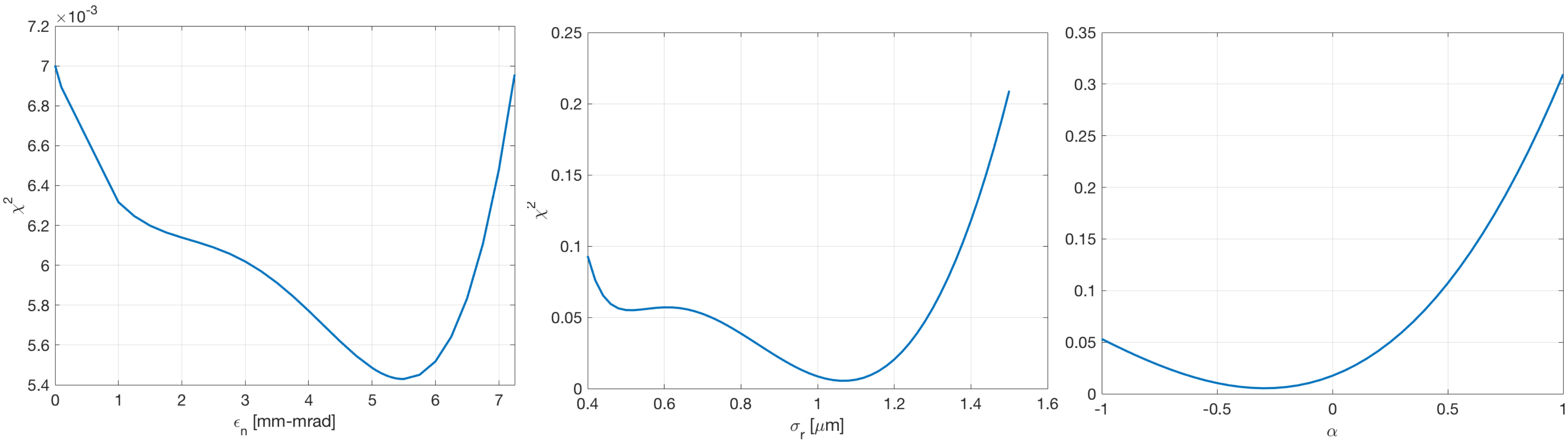

Beams of Various emittance below 21 GeV are very difficult to distinguish

# Conclusions

- An optimization routine is used to estimate the parameters of the beam consistent with the beam size observed in a high resolution spectrometer
- Emittance of injected beam from E217 is estimated at 5 mm-mrad, much smaller than the drive beam
- The imaging spectrometer can be used to measure the emittance growth of the witness beam in the two bunch experiment in FACET II to accuracy of tens of percent
- The more mismatched the beam is, the harder it is to measure emittance growth

The End

# Checking Optimization Variation




Variation of $\chi^2$ attains a minimum for all three parameters

$$\chi^2 = \sum \frac{(\sigma_{opt} - \sigma_{obs})^2}{\sigma_{obs}{}^2}$$

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